

Lehrstuhl für Netzwerktheorie und Signalverarbeitung  
der Technischen Universität München

# **Wireless MIMO Systems – Models, Performance, Optimization**

Michel T. Ivrláč

Vollständiger Abdruck der von der Fakultät für Elektrotechnik und Informations-  
technik der Technischen Universität München zur Erlangung des akademischen  
Grades eines

Doktor-Ingenieurs

genehmigten Dissertation.

Vorsitzender: Univ.-Prof. Dr.-Ing. Joachim Hagenauer

Prüfer der Dissertation:

1. Univ.-Prof. Dr. techn. Josef A. Nossek
2. Prof. Dr. Dr. h.c. James L. Massey,  
em. Eidgenössische Technische Hochschule,  
Zürich, Schweiz

Die Dissertation wurde am 30.11.2004 bei der Technischen Universität München  
eingereicht und durch die Fakultät für Elektrotechnik und Informationstechnik am  
18.02.2005 angenommen.



Berichte aus dem Lehrstuhl für Netzwerktheorie und  
Signalverarbeitung der Technischen Universität München

**Michel Ivrlač**

**Wireless MIMO Systems –  
Models, Performance, Optimization**

Shaker Verlag  
Aachen 2005

**Bibliographic information published by Die Deutsche Bibliothek**

Die Deutsche Bibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data is available in the internet at <http://dnb.ddb.de>.

Zugl.: München, Techn. Univ., Diss., 2005

Copyright Shaker Verlag 2005

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the publishers.

Printed in Germany.

ISBN 3-8322-4390-9

ISSN 1433-1446

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

Internet: [www.shaker.de](http://www.shaker.de) • eMail: [info@shaker.de](mailto:info@shaker.de)

## Acknowledgments

Back in 1998 Univ. Prof. Dr.-techn. Josef A. Nossek offered me a position as a research engineer and teaching assistant at his renown Institute for Circuit Theory and Signal Processing at Munich University of Technology. In being the supervisor of my doctoral thesis he contributed in many ways to my work of the past years. His way of asking the right questions at the right time and his ability to create a working environment and atmosphere that is just perfect for doing research are two of them. Numerous discussions and his careful reading of our published and unpublished work always motivated me. Besides for technical and scientific problems he always had an open ear for human issues, too. I want to express my sincere thanks to Professor Josef A. Nossek to whom I am greatly indebted. Without his commitment and professional attitude this doctoral thesis would not have been possible.

The first time that I saw Professor James L. Massey in real life was in Sorrento, Italy, where he was one of the panelists at the ISIT 2000. His impressive words left a permanent imprint in my memory. The second time that I had the pleasure to meet Professor Massey was in his capacity as the second reviewer of my doctoral thesis. Honestly, I was a little bit worried to have my thesis examined by the critical eyes of such a highly decorated and worldwide renowned researcher. I am greatly indebted to Professor James L. Massey for his interest in my work, his thorough reading of my thesis and the many useful comments that have considerably improved its quality.

During the time when I was working towards the doctoral degree, I had the pleasure to meet a variety of colleagues and friends. I want to thank all of them for making my life more colorful. I also want to thank our secretaries and our technical staff members for keeping the infrastructure of the institute functioning smoothly all the time. Furthermore, I want to thank Univ. Prof. Dr.-Ing. Wolfgang Utschick, especially for the help of bringing my wife into my life.

From the bottom of my heart, I thank my parents who have given birth to my life. They supplied me with all the necessary ingredients that have lead to the human being that I am today. I am greatly indebted to their support and love in all those, sometimes eventful years.

The love of my wife has always been an endless source of inspiration and happiness. My deep thanks go to her for together creating our tiny, happy family which became even more complete 18 month ago, when our son was born.

This work is dedicated to my wife Lai U and our son Gallus.

Munich, July 2005

Michel T. Ivrlač



# Contents

<b>1. Introduction</b>	<b>1</b>
1.1 Motivation and Major Contributions	1
1.1.1 Modeling	2
1.1.2 Performance Measures	5
1.1.3 System Optimization	7
1.2 Thesis Overview	8
1.3 Notational Issues	13
<b>2. Information and Communication</b>	<b>17</b>
2.1 Foundations	19
2.1.1 Entropy	19
2.1.2 Mutual Information	20
2.1.3 The Chain Rule of Information Theory	22
2.1.4 Continuous Entropy	23
2.1.5 Discrete Memoryless Channel	23
2.1.6 Continuous Memoryless Channel	24
2.1.7 Channel Capacity	24
2.1.8 The CAWGN Channel	24
2.1.9 General Channel Encoding System	25
2.1.10 Noisy Channel Coding Theorem	26
2.1.11 Successive Interference Cancellation	27
2.1.12 Bandwidth in Continuous Channels	28
2.1.13 Noise	29
2.1.14 Bandwidth Efficiency	30
2.1.15 The Shannon Limit	31
2.2 On Transmit Channel Information	31
2.3 Time varying Channels	32
2.3.1 Slowly Varying Channel	33
2.3.2 Ergodic and Instantaneous Channel Capacity	34
2.3.3 Temporal Transmit Power Loading	34
2.3.4 Constant Transmit Power	36
2.3.5 Outage Capacity	36
2.3.6 Piecewise Constant Independent (PWCI) Channel	37
2.3.7 Block Fading (BF) Channel	38
2.3.8 Sample Mean Outage Capacity	39
2.4 Multichannel Systems	39
2.4.1 Power and Bandwidth Efficiency	40
2.4.2 Multiple Parallel Channels	41
2.4.3 Modes of Operation	43
2.4.4 The COM-Chart	45

<b>3. Wireless Channel</b>	<b>47</b>
3.1 Electromagnetic Foundations . . . . .	49
3.1.1 Maxwell Equations in Free Space . . . . .	49
3.1.2 Planar Transversal Waves . . . . .	49
3.1.3 Far field Approximation . . . . .	51
3.1.4 Energy and Power . . . . .	52
3.1.5 Nonfree space Propagation . . . . .	54
3.1.6 Receive and Transmit Antennas . . . . .	54
3.2 Modulation and Complex Baseband . . . . .	55
3.2.1 Carrier Modulation . . . . .	55
3.2.2 Complex Baseband . . . . .	59
3.2.3 Signal Power and Complex Baseband Noise . . . . .	60
3.3 Channel Model . . . . .	62
3.3.1 Optimum Transmit- and Receive Filtering . . . . .	62
3.3.2 Tapped Delay Line Model . . . . .	66
3.3.3 Sub-optimum Receive Filtering . . . . .	68
3.3.4 Block Processing . . . . .	69
3.3.5 Frequency Domain Representation . . . . .	70
3.3.6 Periodically Time Varying CAWGN Channel . . . . .	72
3.3.7 Capacity of the Time Invariant Multipath Channel . . . . .	73
3.4 Mobility . . . . .	74
3.4.1 Doppler Shift and Aberration . . . . .	75
3.4.2 Doppler Spread and Small scale Fading . . . . .	77
3.4.3 Coherence Time and Decorrelation Time . . . . .	80
3.4.4 Rayleigh and Ricean Fading . . . . .	80
3.4.5 $(\epsilon, \bar{\tau})$ - Applicability of the Channel Model . . . . .	81
3.4.6 Broadband and Narrowband Channel . . . . .	83
3.4.7 On Block Length and Overhead in Broadband Channels . . . . .	84
<b>4. MIMO Channel</b>	<b>85</b>
4.1 Antenna Arrays . . . . .	87
4.1.1 Signal Sampling in Space . . . . .	87
4.1.2 Narrowband Assumption . . . . .	88
4.1.3 Array Steering Vector . . . . .	88
4.1.4 Uniform Linear Array . . . . .	89
4.1.5 Spatial Frequency . . . . .	89
4.1.6 Transmit Beamforming and Antenna Gain . . . . .	92
4.2 MIMO Channel . . . . .	95
4.2.1 Impulse Response . . . . .	95
4.2.2 Tapped Delay Line . . . . .	97
4.2.3 Frequency Domain Representation . . . . .	98
4.2.4 Time variation . . . . .	101
4.3 Stochastic Channel Model . . . . .	101
4.3.1 Discrete Spatial Multipath Structure . . . . .	101
4.3.2 Rayleigh Fading Channel Model . . . . .	104
4.3.3 Independent Receive and Transmit Fading . . . . .	105



4.3.4	Types of Independent Correlated Fading . . . . .	109
4.3.5	Frequency Flat Channel . . . . .	112
4.3.6	Non-fading Line of Sight . . . . .	112
4.3.7	Random Channel Generation . . . . .	112
<b>5.</b>	<b>MIMO Performance Measures</b> . . . . .	<b>115</b>
5.1	Mutual Information . . . . .	117
5.2	Channel Capacity . . . . .	120
5.2.1	Instantaneous Transmit Channel State Information . . . . .	120
5.2.1.1	Information Theoretic Derivation . . . . .	120
5.2.1.2	Signal Processing Point of View . . . . .	124
5.2.1.3	Rank Deficient Channels . . . . .	127
5.2.2	No Transmit Channel Knowledge . . . . .	129
5.2.3	Long Term Transmit Channel Knowledge . . . . .	131
5.2.3.1	MIMO Eigenbeamforming . . . . .	131
5.2.3.2	Average Correlation Matrices . . . . .	134
5.2.3.3	Transmit Signal Processing and Receive Interference Cancellation . . . . .	137
5.2.4	Why Long Term Transmit Channel Knowledge? . . . . .	140
5.2.5	Uncorrelated vs. Semi-correlated MIMO Channels . . . . .	141
5.3	Cutoff-Rate . . . . .	142
5.3.1	QAM Modulation Alphabet . . . . .	143
5.3.2	Gallager Exponent and Cutoff Rate Theorem . . . . .	145
5.3.3	Cutoff-Rate for Linearly Modulated Memoryless MIMO Channels . . . . .	146
5.3.4	Semi-Correlated Fading . . . . .	148
5.4	Coded Packet Error Probability . . . . .	149
5.4.1	Packet Error Probability in a Fading Channel . . . . .	151
5.4.2	Packet Error Probability with Interleaving . . . . .	151
5.5	Antenna Gain . . . . .	153
5.5.1	Instantaneous Transmitter Channel Knowledge . . . . .	154
5.5.2	Long-Term Transmit Channel Knowledge . . . . .	155
5.5.3	No Transmitter Channel Knowledge . . . . .	156
5.5.4	Antenna Gain and Channel Capacity . . . . .	156
5.5.5	Long-term vs. Average Instantaneous Antenna Gains . . . . .	158
5.6	Multiplexing Gain . . . . .	159
5.6.1	Instantaneous Transmit Channel Knowledge . . . . .	159
5.6.2	Long-term Transmit Channel Knowledge . . . . .	161
5.7	Diversity Gain . . . . .	162
5.7.1	Cumulative Probability Distribution . . . . .	162
5.7.2	Diversity and Diversity Order . . . . .	163
5.7.3	Diversity Measure . . . . .	165
5.7.4	Long-term Diversity Gain . . . . .	168
5.7.5	The Correlation Measure . . . . .	169
5.8	Fundamental Gain Trade-off . . . . .	171

<b>6. MIMO System Optimization</b>	<b>175</b>
6.1 Layered System Design	177
6.1.1 Layered System Overview	177
6.1.2 Standard Layered Design Paradigm	179
6.1.3 Cross Layer Design Paradigm	179
6.2 Cross Layer Optimization	180
6.2.1 Operating Points and Operating Modes	181
6.2.2 Top-Down and Bottom-up Approach	182
6.2.3 Set-based Framework Formalism	183
6.3 Quality of Service	186
6.3.1 File Transfer Application	186
6.3.2 Video Streaming Application	188
6.4 Cross Layer Optimization Examples	191
6.4.1 Single-user Top-Down Approach	191
6.4.2 Multi-user Bottom-Up Approach	196
<b>7. Summary</b>	<b>205</b>
<b>Appendix</b>	<b>207</b>
A1 Entropy of Information Sources with Memory	207
A2 Rate Distortion of a Memoryless Gaussian Source	208
A3 Entropy Maximizing Distribution	209
A4 Cutoff Rate for Linear Modulation	211
A5 Arithmetic and Geometric Mean Inequality	213
A6 Hadamard Inequality for positive definite Matrices	213
A7 A Matrix Rotation Identity	214
A8 Lower Bound on the Solution of $(1 + x/r^2)^r > 1 + x$	215
A9 Diversity Measure	216
A10 Diversity Measure (Cont.)	218
A11 The Lorentz Transform	221
A12 Relativistic Doppler Effect	224
A13 Relativistic Beamforming	226
A14 Noise Whitening FIR-filter	228
A15 Delay-Spread and Coherence-Bandwidth	231
A16 Source Distortion in Compressed Video Streaming	234
A17 Distortion due to Packet Loss in Compressed Video Streaming	236
<b>Bibliography</b>	<b>247</b>