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Sankaranarayanan Subramanian

Tube-enhanced Multi-stage Model Predictive Control

Robust State and Output Feedback Control

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Declaration of authorship

I, Sankaranarayanan SUBRAMANIAN, declare that this thesis titled, "Tube-enhanced Multi-stage Model Predictive Control: Robust State and Output Feedback Control" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
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Dortmund, 03. August 2020

Previously published material

Parts of the results that are presented in this work and that are used in the context of this dissertation were published beforehand or are currently submitted for publication. These publications are indicated by references throughout the thesis and they are listed below with references to the respective chapters. For each publication, Sankaranarayanan Subramanian, in the following called *the author*, provides a statement about the extent of the contribution.

The classification of the contribution of the author is defined as

largely – a contribution is classified as *largely*, if most of the work (scientific, technical, and writing) was done (solely) by the author.

equally – a contribution is classified as *equally*, if the author and the coauthor(s) contributed to the work to the same extent.

partly – a contribution is classified as *partly*, if the author contributed to the work (scientific, technical, and writing) only to a certain extent.

Notice, however, that scientific work is usually done in a joint effort of a research group, where ideas evolve in discussions and debates about scientific results and methodologies. Thus, a clear distinction of the individual contribution is often impossible. The following comments were made to the best knowledge of the author.

Further publications of the author that were not directly reused in the context of this work can be found at the end of this thesis on page 277.

Chapter	Statement	Reference(s)
Chapter 2	Published in	Subramanian et al. (2018a)
Chapter 3	Submitted for publication in	Subramanian et al. (2020)
Chapter 4	Basic ideas published in	Subramanian et al. (2018b)
	Stability proof of the primary controller published in Section 4 of	Lucia et al. (2020)
Chapter 6	Basic ideas published in	Subramanian et al. (2016a)
	Published in	Subramanian et al. (2017a)
Chapter 7	Time-invariant version published in	Subramanian et al. (2017b)
Chapter 8	Scheme analyzed, simulation study performed in	Abuel-nour (2018)
	Published in	Subramanian et al. (2019)
Chapter 10	The output feedback results are published in	Subramanian et al. (2018b)
Chapter 11	All the results provided in this chapter were done as a part of	Abdelsalam (2018)
	Some results are also published in the context of comparison with a simplified formulation	Abdelsalam et al. (2020a)

Peer reviewed journal publications

(Lucia et al., 2020) Lucia, S.; Subramanian, S.; Limon, D.; Engell, S. Stability properties of multi-stage nonlinear model predictive control. *Systems & Control Letters* **2020,** *143,* 104743

The author conceptualized and wrote the ISS stability section (Section 4) in the publication *largely*. The author also contributed in the implementation of the example *equally*. S. Lucia developed the methodology, the ISpS stability and wrote the overall manuscript *largely*. S. Lucia also contributed in the implementation of the example *equally*. D. Limon contributed *partly* with respect to the stability proofs. S. Engell was involved in the development of the methodology *partly*. All the authors contributed to the writing, revision, and proof reading.

(Subramanian et al., 2020) Subramanian, S.; Lucia, S.; Paulen, R.; Engell, S. Tube-enhancedMulti-stageMPC for Flexible Robust Control of Constrained Linear Systems with Additive and Parametric Uncertainties. *International Journal of Robust and Nonlinear Control* **2020,** Accepted for publication

Research, method development, theory simulation study, and writing were done by the author *largely*. S. Lucia, R. Paulen, and S. Engell were involved in the development of the method *partly*. All the authors contributed to the writing, revision, and proof reading.

Peer reviewed conference papers

(Abdelsalam et al., 2020a) Abdelsalam, Y.; Subramanian, S.; Engell, S. A Simplified Implementation of Tube-Enhanced Multi-Stage NMPC.. *21st IFAC World Congress* **2020a,** Accepted for publication

Research, method development, theory and writing were done by Y. Abdelsalam *largely*. The author and S. Engell contributed *partly* to the development of the methodology. All the authors contributed to the writing and proof reading.

(Subramanian et al., 2019) Subramanian, S.; Aboelnour, M.; Engell, S. Robust Tube-enhanced Multi-stage Output Feedback MPC for Linear Systems with Additive and Parametric Uncertainties. *Proc. of the 18th European Control Conference (ECC)* **2019,** 331–336

Research, method development, theory were done by the author *largely*. Contributed in writing *equally*. M. Abuel-nour contributed *equally* in writing. M. Abuel-nour contributed *largely* in simulation studies. S. Engell was involved in the development of the method *partly*. All the authors contributed to the proof reading.

(Subramanian et al., 2018b) Subramanian, S.; Lucia, S.; Engell, S. A synergistic approach to robust output feedback control: Tube-based multi-stage NMPC.. *IFAC-PapersOnLine* **2018b,** *51* (18), 500–505

Research, method development, theory, simulation study and writing were done by the author *largely*. S. Lucia and S. Engell were involved in the development of the method *partly*. All the authors contributed to the proof reading.

(Subramanian et al., 2018a) Subramanian, S.; Lucia, S.; Birjandi, S. A. B.; Paulen, R.; Engell, S. A Combined Multi-stage and Tube-based MPC Scheme for Constrained Linear Systems. *IFAC-PapersOnLine* **2018a,** *51* (20), 481–486

Research, method development, theory, simulation study and writing were done by the author *largely*. S. Lucia, R. Paulen, S. Birjandi and S. Engell were involved in the development of the method *partly*. S. Birjandi also contributed to the simulation study *partly*. All the authors contributed to the proof reading.

(Subramanian et al., 2017a) Subramanian, S.; Lucia, S.; Engell, S. An Improved Output Feedback MPC scheme for Constrained Linear Systems. *IFAC-PapersOnLine* **2017a,** *50* (1), 15506–15511

Research, method development, theory, simulation study and writing were done by the author *largely*. S. Lucia and S. Engell were involved in the development of the method *partly*. All the authors contributed to the proof reading.

(Subramanian et al., 2017b) Subramanian, S.; Lucia, S.; Engell, S. A novel tube-based output feedback MPC for constrained linear systems. *Proc. of the American Control Conference (ACC)* **2017b,** 3060–3065

Research, method development, theory, simulation study and writing were done by the author *largely*. S. Lucia and S. Engell were involved in the development of the method *partly*. All the authors contributed to the proof reading.

(Subramanian et al., 2016a) Subramanian, S.; Lucia, S.; Engell, S. A Non-conservative Robust Output Feedback MPC for Constrained Linear Systems. *Proc. of the 54th IEEE Conference on Decision and Control* **2016a,** 2333–2338

Research, method development, theory, simulation study and writing were done by the author *largely*. S. Lucia and S. Engell were involved in the development of the method *partly*. All the authors contributed to the proof reading.

Master's Thesis

(Abuel-nour, 2018) Abuel-nour, M. Robust Output Feedback Model Predictive Control of Constrained Linear Systems. MA thesis, TU Dortmund, 2018

The author provided the robust formulation of the linear tube-enhanced multi-stage MPC scheme. M. Abuel-nour performed analysis of the formulation, implemented examples, performed simulation study *largely*. M. Abuelnour also developed the adaptive formulation of the scheme *largely*. The author proofread the thesis and provided supervision.

(Abdelsalam, 2018) Abdelsalam, Y. Robust Output Feedback Nonlinear Model Predictive Control. MA thesis, TU Dortmund, 2018

The author provided the robust formulation of the tube-enhanced multi-stage NMPC scheme. Y. Abdelsalam performed analysis of the formulation, implemented examples, performed simulation study *largely*. Y.Abdelsalam also studied the theoretical aspects of the scheme, and proposed the simplified and adaptive formulations of the scheme *largely*. The author proofread the thesis and provided supervision.

Abstract

The principal theme of the thesis is addressing the challenge of the trade-off between optimality and complexity in the field of robust Model Predictive Control (MPC). Different robust MPC formulations are proposed that guarantee rigorous constraint satisfaction, recursive feasibility, and robust asymptotic stability for linear and nonlinear systems with additive and parametric uncertainties under standard assumptions. For linear systems with full-state feedback, a basic formulation and an advanced formulation of Tube-Enhanced Multi-Stage (TEMS) MPC are presented. The basic formulation avoids the problem of growth in problem complexity concerning less significant uncertainties by computing a constant feedback law offline and improves the performance of the controller by calculating a sequence of control policies for significant uncertainties modeled on a scenario-tree. The advanced formulation provides users with additional options to limit the rapid growth of problem complexity beyond a certain prediction step called robust horizon with the help of affine feedback policies. It results in a linear increase in problem complexity with respect to the prediction horizon beyond the robust horizon. For nonlinear systems, the proposed TEMS Nonlinear MPC (NMPC) scheme implements two multi-stage controllers hierarchically to achieve robustness against uncertainties.

Novel robust output feedback formulations are proposed that can work with any estimation scheme that can provide an estimate along with an error bound. The concept of substitute estimates is introduced to obtain control policies for the linear output feedback case using the multi-stage, the tube-based, and the TEMS MPC schemes. For nonlinear systems, different output feedback formulations using the proposed TEMS NMPC scheme are proposed and analyzed. The resulting schemes can handle estimation errors in addition to plant-model mismatches while being less conservative with manageable complexity. The advantages of the proposed schemes are demonstrated using two case studies: a semi-batch industrial polymerization reactor and a continuous Williams-Otto reactor.

Kurzfassung

Als grundlegendes Thema behandelt diese Dissertation die Schwierigkeit des Kompromisses zwischen Optimalität und Komplexität im Bereich der robuste Modellprädiktiven Regelung (MPC). Verschiedene robusten MPC-Formulierungen werden vorgeschlagen, die rigorose Erfüllung der Nebenbedingungen, rekursive Zulässigkeit und robuste, asymptotische Stabilität für lineare und nichtlineare Systeme mit additiven und parametrischen Unsicherheiten unter Standardannahmen garantieren. Für lineare Systeme mit vollständiger Zustandsrückführung werden eine grundsätzliche Formulierung und eine erweiterte Formulierung von Tube-Enhanced Multi-Stage (TEMS) MPC vorgestellt. Die grundsätzliche Formulierung vermeidet das problematische Wachstum der Problemkomplexität durch wenig signifikante Unsicherheiten durch die offline Berechnung einer konstanten Rückführungsregel und verbessert die Leistung des Reglers durch die Aufstellung einer Abfolge von Regelungsstrategien für signifikante Unsicherheiten, die in einem Szenariobaum modelliert werden. Die erweiterte Formulierung bietet dem Anwender zusätzliche Möglichkeiten zur Einschränkung des schnellen Wachstums der Komplexität ab einem bestimmten Vorhersageschritt durch Einführung affiner Rückführungsregeln. Das Ergebnis ist ein lineares Wachstum der Problemkomplexität abhängig vom Vorhersagehorizont über den robusten Horizont hinaus. Für nichtlineare Systeme implementiert das vorgeschlagene TEMS nichtlineare MPC (NMPC) Schema zwei Multi-stage Regler hierarchisch um Robustheit gegen Unsicherheiten zu erreichen.

Neue robuste Formulierungen der Ausgaberückführung werden vorgeschlagen, die mit einem beliebigen Schema zur Schätzung verwendet werden können, das einen Schätzwert zusammen mit einer Fehlerschranke liefert. Das Konzept von Ersatzschätzwerten wird eingeführt, um Regelungsstrategien für Fall der linearen Ausgaberückführung unter Verwendung des multi-stage, des tube-basierten und des TEMS-MPC Schemas. Für nichtlineare Systeme werden verschiedene

Formulierungen zur Ausgaberückführung mit dem vorgeschlagenen TEMS MPC Schema vorgeschlagen und analysiert. Die resultierenden Schemata können Schätzungsfehler zusätzlich zu Anlagen-Modell-Abweichungen handhaben und sind gleichzeitig weniger konservativ bei der bewältigbaren Komplexität. Die Vorteile der vorgeschlagenen Schemata werden in zwei Fallstudien gezeigt: Ein Semi-Batch industrieller Polymerisationsreaktor und ein kontinuierlicher Williams-Otto Reaktor.

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Contents