

FUNDAMENTALS OF ROBUSTNESS ANALYSIS

**Mustafa Berber
Petr Vaníček
Peter Dare**

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PREFACE

Studies about Robustness Analysis at University of New Brunswick started in early 1980s. Since then many researchers have contributed to developments about Robustness Analysis. These researchers also collaborated with researchers at University of Calgary and Geodetic Survey Division of Canada under contracts on Robustness Analysis.

This study addresses the developments that have been made on Robustness Analysis.

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