

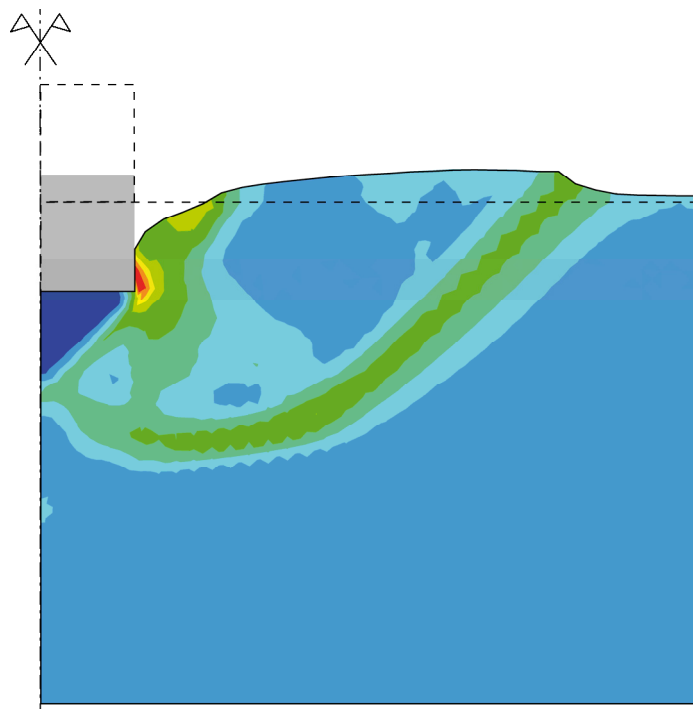
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An Arbitrary Lagrangian-Eulerian Method for  
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Daniel Aubram

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# **An Arbitrary Lagrangian-Eulerian Method for Penetration into Sand at Finite Deformation**

vorgelegt von  
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# Preface by the Editor

The design of foundations and geotechnical structures and the prediction of their deformation and mechanical response based on numerical simulations require detailed knowledge of the local state of the soil as well as sufficiently realistic models for the nonlinear soil behavior. This is particularly true for sand whose behavior depends among others on the stress state, density state, and loading history. The local state is a result of loading history, hence the construction processes of structural elements must generally be taken into account. Common numerical models in geotechnical engineering, however, consider the construction process as a simplified staged construction by assuming an initial state of the soil and taking the structural elements as wished-in-place. The research work reported here is a step towards more realistic modeling.

The present work submitted by Daniel Aubram is concerned with the penetration into sand as a typical geotechnical process and its numerical simulation by using the finite element method (FEM). Apart from the highly nonlinear mechanical behavior of sand and the presence of non-stationary material interfaces this challenging problem is characterized by large local material deformations for which the classical Lagrangian and Eulerian approaches are inapplicable. In this work a new arbitrary Lagrangian-Eulerian (ALE) method is developed and implemented in which the material and mesh deformations are uncoupled. Its unique features are an advanced hypoplastic rate constitutive equation for sand and an efficient and robust optimization-based algorithm to ensure a quality FE mesh during the entire calculation. The method is validated by results from the literature and specifically conducted experimental model tests.

This thesis addresses a wide range of topics of continuum mechanics, soil mechanics, and computational mechanics. Daniel Aubram has acquired advanced knowledge of all these fields during his comprehensive research work. Finally, he achieved both unusual depth and coherence of his thesis by means of a modern mathematical language and consistent notation. Most of the background needed is in the main text or can be found in the noteworthy appendix. Beyond that, Daniel Aubram has set up the necessary computer code and supervised the experimental program.

Parts of this research work have been carried out under the financial support from the German Research Foundation (DFG) through grants SA 310/21-1 and SA 310/21-2 which is gratefully acknowledged.

Stavros A. Savidis  
Berlin, December 2013



# Preface by the Author

The work presented in this thesis was carried out between 2006 and 2011 when I was a research associate at the Chair of Soil Mechanics and Geotechnical Engineering at the Technische Universität Berlin.

I would like to express my deepest gratitude to my advisor and principal reviewer Prof. Dr.-Ing. Stavros A. Savidis for giving me the opportunity to work on this interesting topic and for the freedom to develop and implement my own ideas while continuously guiding and supporting me during all stages of my research work. Moreover, I thank Prof. Savidis for his encouragement and support in presenting the results of my research work on several national and international conferences. I am also indebted to Prof. Dr.-Ing. Peter Wriggers from Leibniz Universität Hannover for his interest in my work and preparing the second review of my thesis. Being a well-known expert in the field of computational mechanics he enriched the area of research addressed by this work and its interdisciplinary aspects. I am grateful to my former colleague at TU Berlin and third reviewer Prof. Dr.-Ing. Frank Rackwitz from Ostbayerische Technische Hochschule Regensburg for several fruitful discussions and helpful suggestions. In addition, I wish to acknowledge Prof. Dr.-Ing. Yuri Petryna for his feedback and taking the chair of the doctoral committee.

I would like to thank my former and present colleagues at the Chair of Soil Mechanics and Geotechnical Engineering at TU Berlin for the very pleasant working atmosphere. Many of them were always willing to discuss ideas and to answer questions, first of all Dipl.-Ing. Winfried Schepers, Dr.-Ing. H. Ercan Taşan, Dipl.-Ing. Ralf Glasenapp, and Dipl.-Ing. Marcel Ney. Moreover, I thank the technical staff of our geotechnical laboratory, especially Mr. Rüdiger von König and Mr. Harald Lorenz, for helping to realize the experimental model tests. Preliminary work for the experimental part of this thesis was carried out at the institute by several undergraduate students within the scope of their final theses, which is also gratefully acknowledged.

Finally, I am deeply grateful to my family for unconditionally supporting me during my whole life, especially my parents Bärbel and Stefan Aubram. Above all, I thank my wife Janine for her constant patience and support in successfully finishing this thesis and for being a loving mother to our son Konstantin.

Daniel Aubram  
Berlin, December 2013





# Abstract

The penetration into sand is one of the most challenging problems in soil mechanics and its numerical simulation, particularly by the widely-used finite element method (FEM), still poses a great challenge. In order to overcome the problems associated with the classical Lagrangian and Eulerian formulations of FEM, an arbitrary Lagrangian-Eulerian (ALE) method particularly suitable for plane and axisymmetric penetration into sand is developed in this thesis based upon the theoretical foundations.

The developed ALE method applies an operator-split which breaks up solution of the governing equations over a time step into a Lagrangian step, a mesh regularization step, and a transport step. The operator-split makes implementation into existing Lagrangian FE codes possible, which is shown using the example of ANSYS. A unique feature of the ALE method is its combination with an advanced hypoplastic rate constitutive equation for sand which enables realistic prediction of stress and density changes within the soil even at large deformations. In addition, an optimization-based algorithm for mesh regularization is developed in order to smooth out the non-convexly distorted mesh regions that occur below a penetrator. The ALE method is verified and validated by benchmarks, basic initial boundary value problems, and by specifically conducted penetration tests in chambers filled with sand.

**Keywords:** arbitrary Lagrangian-Eulerian; large deformations; finite element method; penetration; pile; sand; soil mechanics; continuum mechanics; computational mechanics; geotechnical engineering



# Zusammenfassung

Die Penetration in Sand zählt zu den kompliziertesten Problemstellungen in der Bodenmechanik, und ihre numerische Simulation insbesondere mit der weit verbreiteten Finite Elemente Methode (FEM) stellt bis heute eine große Herausforderung dar. Um die Probleme im Zusammenhang mit den klassischen Lagrange und Euler Formulierungen der FEM zu überwinden, wird in der vorliegenden Arbeit eine allgemeine Lagrange-Euler (engl.: arbitrary Lagrangian-Eulerian, kurz: ALE) Methode aus den theoretischen Grundlagen heraus speziell für die ebene und axialsymmetrische Penetration in Sand entwickelt.

Die entwickelte ALE Methode basiert auf einer Operator-Spaltung, welche die Lösung der maßgeblichen Gleichungen über ein Zeitinkrement aufteilt in einen Lagrange Schritt, einen Schritt der Netzregularisierung und einen Transportschritt. Die Operator-Spaltung gestattet die Implementierung in bestehende Lagrange FE Programmsysteme, was am Beispiel von ANSYS erläutert wird. Ein Alleinstellungsmerkmal der ALE Methode ist ihre Kombination mit einem hochentwickelten hypoplastischen Materialmodell für Sand, das wirklichkeitsnahe Prognosen der Spannungs- und Dichteänderungen im Boden auch bei großen Verformungen ermöglicht. Ein optimierungsbasierter Algorithmus zur Netzregularisierung wird darüber hinaus entwickelt, um die unterhalb eines Eindringkörpers auftretenden nicht-konvex verzerrten Netzregionen zu glätten. Die ALE Methode wird anhand von Benchmarks, grundlegenden Anfangsrandwertproblemen und eigens durchgeführten Eindringversuchen in sandbefüllten Versuchskammern verifiziert und validiert.

**Schlagnworte:** Arbitrary Lagrangian-Eulerian; große Verformungen; Finite Elemente Methode; Penetration; Pfahl; Sand; Bodenmechanik; Kontinuumsmechanik; Numerische Mechanik; Geotechnik



*To my family*

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