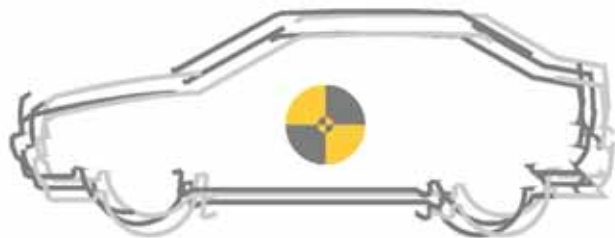


UNIVERSITY OF AGDER  
FACULTY OF ENGINEERING AND SCIENCE

# DEVELOPMENT OF MATHEMATICAL MODELS FOR VEHICLE TO POLE COLLISION

BACHELOR'S THESIS



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Grimstad, Norway

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Witold Pawlus, Jan Eivind Nielsen



## Abstract

This work is devoted to mathematical modeling of vehicle to pole collision. By the analysis of the acceleration signal recorded during the crash test, changes of vehicle's velocity and displacement are obtained. Comparative analysis of those responses is done for wideband (unfiltered) and filtered data. Significant differences are observed, therefore the second dataset, as the proper one, is used for the rest of the project. Methods for viscoelastic modeling are presented. We investigate four basic models which consist of springs, dampers and masses in different arrangements. We introduce ways for obtaining parameters of those models based on the crash pulse analysis. After the models are established and simulated, their responses are compared with each other as well as with the original car's crush. From that comparison we conclude which model is the most suitable for vehicle to pole collision simulation. As an interesting addition to this project, the bases of vehicle to occupant modeling together with manners of occupant's crash severity assessment are elaborated.



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