

Lehrstuhl für Baumechanik der Technischen Universität München

PLASTIC SHAPE FUNCTIONS OF PLATE SYSTEMS –
Reducing Modal DOF for Stochastic Nonlinear Dynamics
of Large Scale Plate Systems

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Abstract

Plate systems which excited by stochastic dynamic loadings with the consideration of the material non-linearity is investigated. To describe the arisen plastifications, a 2D hysteretic material model is used. In order to reduce the number of modal degrees of freedom in the numerical analysis within the time domain as by the elastic dynamics, the plastic shape functions are introduced, which are specifically fitted to the plastic parts of curvatures resulted from the yielding and are systematically developed by means of the FEM. Due to the high reduction capability, 10,000 Monte-Carlo-Simulations can be performed for instance on a flat slab with about 30,000 FE-DOF.

Keywords

Plate system, flat slab, 2D hysteretic material model, FEM, BFS/Schäfer plate element, modal extension, Plastic Shape Function, stochastic dynamics, Monte Carlo Simulation

Plastische Formfunktionen der Plattensysteme – Reduzieren modale DOF für stochastische nichtlineare Dynamik der großen Plattensysteme

Zusammenfassung

Untersucht werden Platten unter stochastischen dynamischen Anregungen mit Berücksichtigung der Material-Nichtlinearität. Zur Beschreibung der auftretenden Plastizierungen wird ein 2D hysteretisches Materialmodell verwendet. Um die Anzahl der modalen Freiheitsgrade der numerischen Analyse im Zeitbereich ähnlich wie in der elastischen Dynamik reduzieren zu können, werden “Plastische Formfunktionen” eingeführt, die speziell den plastischen Krümmungsanteilen angepasst sind und mittels FEM systematisch entwickelt werden. Das hohe Reduktionspotential ermöglicht z.B. 10.000 Monte-Carlo-Simulationen an einer Flachdecke mit ca. 30.000 FEM-Freiheitsgraden.

Schlüsselwörter

Plattensystem, flache Decke, 2D hysteretische Materialmodell, FEM, BFS/Schäfer Plattenelement, modale Erweiterung, Plastische Formfunktionen, stochastische Dynamik, Monte Carlo Simulation

Vorwort

Die vorliegende Arbeit entstand am Lehrstuhl für Baumechanik der Technischen Universität München während meiner selbständigen Tätigkeit in den Jahren 2002-2006.

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München, im März 2007

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Notation

Abbreviation

2D-HMiPSP	two-dimensional hysteretic model in principal stress plane
DMCS	direct Monte-Carlo simulation
DOF	degrees of freedom
ESL	equivalent statistical linearization
MCS	Monte-Carlo simulation
MDOF	multi degrees of freedom
NSD	nonlinear stochastic dynamics
PSF	plastic shape function
SDOF	single degree of freedom
VYD	value of yielding determinate of the 2D-HMiPSP

Latin Alphabet

$B_{x,i}, B_{y,i}, B_{xy,i}$	interpolation functions of curvatures
\mathbf{C}_e	element damping matrix
E	Young's modulus
E^t	tangential stiffness or post-yielding strength
f_B	body force
f_S	surface force
f_T	D'Alembert's inertial force
$F(\sigma_{ij})$	yield function
G	shear modulus
h	thickness of plate
$H(t)$	Heaviside function
$\mathbf{H}_{ex}, \mathbf{H}_{ey}, \mathbf{H}_{exy}$	vectors of accessory hysteretic equations
J'_1, J'_2, J'_3	invariants of deviatoric stress tensor
K^P	plastic modulus
K_b	bending stiffness of the plate
\mathbf{K}_e	element stiffness matrix of elasticity
$\mathbf{K}_{ex}, \mathbf{K}_{ey}, \mathbf{K}_{exy}$	element stiffness matrices of hysteresis
m_x, m_y, m_{xy}	bending and twisting moments per unit length
\bar{M}_p	plastic limit of moment
\bar{M}_y	elastic limit of moment

\mathbf{M}_e	element mass matrix
$N(x, y)$	shape function of deflection
$\tilde{N}(x, y)$	interpolation functions of variable Y
$\tilde{\mathbf{p}}(t)$	vector of stochastic loading process
$\bar{p}(x, y, t)$	dynamic area load
$\mathbf{P}_e(t)$	equivalent element nodal forces
S_{ij}	deviatoric stress components
\mathbf{S}	deviatoric stress tensor
$\mathbf{S}(\omega)$	spectral density function
$u(x, y, z)$	elongations of plate in x-direction
δU	vector of virtual displacement
$v(x, y, z)$	elongations of plate in y-direction
$w(x, y, t)$	deflections of plate
w_i^*	generalized degree of freedom
\mathbf{w}_e	vector of the nodal deformations
$\tilde{\mathbf{w}}_e$	vector of nodal accelerations
$\delta W, \delta W_i, \delta W_e$	total, internal and external virtual work
Y	hysteretic variable for moment-curvature correlations
$\mathbf{Y}_{ex}, \mathbf{Y}_{ey}, \mathbf{Y}_{exy}$	vectors of hysteretic variables
z	hysteretic variable of nonlinear restoring forces

Greek Alphabet

α	ratio of post-yielding to preyielding stiffness
α^h	hardening parameters
δ_{ij}	Kronecker delta
ϵ_Y	yield strain
ϵ_{ij}	strain components
ϵ_{ij}^d	deviatoric strain components
ϵ	strain tensor
$\tilde{\kappa}_p$	plastic limit of curvature
$\tilde{\kappa}_y$	elastic limit of curvature
$\kappa_x, \kappa_y, \kappa_{xy}$	bending and twisting curvatures
ν	Poisson's ratio
μ_G, β_G	coefficients of Gumbel-distribution
μ_S, σ_S	mean and standard deviation of simulation results
ω_i	natural frequency
$\psi(x, y)$	rotation angles of cross-sections of plate
ϕ_i	eigenvector
$\psi_p, \tilde{\psi}_p$	plastic shape function, orthogonalized
Φ	mode-shape matrix
ρ	density of material
$\sigma_1, \sigma_2, \sigma_3$	principal stresses
$\sigma_x, \sigma_y, \tau_{xy}$	plane stresses in Cartesian coordinate
σ_Y, σ_Y^0	yield stress, initial yield stress
σ_{ij}	stress components

θ	principal stress angle
$\tilde{\theta}$	principal strain angle
θ_m	equally distributed random phase angle between $[0..2\pi]$
$\boldsymbol{\sigma}$	stress tensor
τ_Y	yield stress of pure shear
ξ	hysteretic variable for stress-strain correlations
ζ_i	damping ratio
α	element damping coefficient