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ITM-Based FSI-Models for Rooms with Absorptive Boundaries

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Abstract

Models for Fluid Structure Interaction (FSI) in room acoustical calculations are used in many different fields of engineering like automotive industry or civil engineering. For simulations of the spatial resolution of the sound field within acoustic cavities very often techniques based on Finite Element formulations are used.

In order to reduce the number of degrees of freedom and therefore the numerical effort, a model reduction method, based on a Component Mode Synthesis (CMS), is applied in this thesis. Macrostructures are assembled out of single substructures applying shape functions at the interfaces. These substructures contain acoustic components like absorbers or resonators. They are calculated separately in the frame of the CMS approach. The acoustic fluid is modeled with the Spectral Finite Element Method (SFEM) and coupled with plate-like compound absorbers at the interfaces via wavenumber- and frequency-dependent impedances using *Hamilton's Principle* and a *Ritz* approach, where phase correct coupling conditions are ensured. The porous foam in the absorber is modeled with the Theory of Porous Media (TPM) and the impedances are calculated with the help of the Integral Transform Method (ITM).

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*dedicated to
Sabine and Maximilian*

Table of Contents

Abstract	III
Acknowledgments	IV
Table of Symbols	IX
1 Introduction	1
1.1 Motivation	1
1.2 State of Research	2
1.3 Modus Operandi and Layout of the Thesis	6
2 Porous Material	8
2.1 Fundamentals of the Theory of Porous Media	9
2.1.1 Volume Fraction Concept	9
2.1.2 Kinematics and Strains	10
2.1.3 Stresses	12
2.1.4 Dissipation of Energy and Interaction Forces	14
2.2 Balance Equations	18
2.2.1 Balance of Mass	18
2.2.2 Balance of Momentum and Moment of Momentum	20
2.3 System of Partial Differential Equations and Fundamental System	21
2.3.1 Helmholtz Decomposition	21
2.3.2 Fourier Transform and Solution	23
2.3.3 Displacements and Stresses in the Transformed Domain	26
3 Elastic Layers and Air	28
3.1 Homogeneous Material	28
3.1.1 System of Partial Differential Equations	28
3.1.2 Solution in the Transformed Domain	28
3.2 Air as Intermediate Layer and Adjacent Structure	31
4 Compound Absorbers	32
4.1 Boundary Conditions and Transition Conditions	35
4.1.1 Lamé-TPM	35
4.1.2 Helmholtz-TPM	36
4.1.3 Helmholtz-Lamé	37
4.1.4 Sommerfeld Radiation Condition	37
4.1.5 Layers on Reflective Walls	39

4.2	System of Equations	39
4.3	Computation of Acoustic Properties	41
4.3.1	Impedance	41
4.3.2	Absorption Coefficient	44
4.4	Numerical Results for Compound Absorbers	46
4.4.1	Validation with Measurements	46
4.4.2	Comparison with the Rayleigh Model	51
4.4.3	Elastic Plates	56
4.4.4	Air Cushions in Front of Porous Sheets	60
4.4.5	Characteristics of Compound Absorbers	63
5	Fluid Structure Interaction	79
5.1	Hamilton's Principle and Rayleigh-Ritz Approach	80
5.2	Component Mode Synthesis	83
5.2.1	Normal Modes	84
5.2.2	Coupling Modes	87
5.3	Coupling with Impedances	96
5.3.1	Lagrangian and Virtual Work	97
5.3.2	Choice of the Trial Functions	104
5.4	System of Equations	112
5.5	Numerical Results for the FSI-problem	114
5.5.1	Application to 1d Structures	114
5.5.2	Application to 2d Structures	120
5.5.3	Application to 3d Structures	131
	Conclusions and Outlook	134
A	Appendix	137
A.1	Vector Calculus	137
A.2	Fourier Transform - Short Summary	140
A.3	Spectral Finite Element Method	141
A.4	Preconditioning	144
A.5	Linear Structural Model for the Kirchhoff Plate	146
A.6	Supplement: Coupling with Impedances	149
A.7	Supplement: Fourier Approximation of the Trial Functions	155
	List of Figures	159
	Bibliography	163
	Index	172

Table of Symbols

General

λ	[m]	wavelength
ω	[rad/s]	natural circular frequency
Ω	[rad/s]	circular frequency of excitation
k_x, k_y	[rad/m]	wavenumbers in x - and y -direction
$Z(k_x, k_y, \Omega)$	[Ns/(m ³)]	wave impedance
U	[Nm]	potential energy
T	[Nm]	kinetic energy
L	[Nm]	Lagrangian
c	[Nm/s]	damping coefficient
D^{Lehr}	[-]	damping ratio
η^D	[-]	damping loss factor
$A - E$	[-]	coefficients in the formulation of the absorber
$\mathcal{A} - \mathcal{D}$	[-]	abbreviations in the formulation of the absorber
$\mathcal{A} - \mathcal{C}$	[-]	coefficients in the FSI-formulation

Technical Acoustics/FSI

ρ_A	[kg/m ³]	density of the air	$(\rho_A = 1.204 \frac{\text{kg}}{\text{m}^3} \text{ at } 20^\circ\text{C})$
c_A	[m/s]	speed of sound of the air	$(c_A = 343.4 \frac{\text{m}}{\text{s}} \text{ at } 20^\circ\text{C})$
k_A	[rad/m]	wavenumber in the air	
Z_0	[Ns/(m ³)]	plane wave impedance	$(Z_0 = 413.5 \frac{\text{Ns}}{\text{m}^3} \text{ at } 20^\circ\text{C})$
r	[-]	reflection factor	
α	[-]	absorption coefficient	
ϱ	[-]	reflection coefficient	
δ	[-]	dissipation coefficient	

λ		vector of <i>Lagrange</i> multipliers
Φ_A	[m ² /s]	velocity potential in the acoustic fluid
\mathbf{v}_A	[m/s]	velocity in the acoustic fluid
\mathbf{u}_A	[m]	displacement in the acoustic fluid
p_A	[N/m ²]	pressure in the acoustic fluid
$\tilde{\Phi}_A$	[m ² /s]	velocity potential in the <i>Fourier</i> -domain
$\tilde{\mathbf{v}}_A$	[m/s]	velocity in the <i>Fourier</i> -domain
$\tilde{\mathbf{u}}_A$	[m]	displacement in the <i>Fourier</i> -domain
\tilde{p}_A	[N/m ²]	pressure in the <i>Fourier</i> -domain
$\psi_n(y, z)$		trial function for the absorber
$\hat{\psi}_n(y, z)$		<i>Fourier</i> approximation of $\psi_n(y, z)$

Theory of Porous Media

ϕ_α		denotes the constituent α ($\alpha = S$ for the solid- and $\alpha = G$ for the gas-phase)
dv_α	[m ³]	partial volume element of the constituent α
dv	[m ³]	total volume element
B_α		domain of the constituent α
n_α	[-]	volume fraction of the constituent α
$\rho_{\alpha R}$	[kg/m ³]	macroscopic real density
ρ_α	[kg/m ³]	macroscopic partial density
\mathbf{x}	[m]	position vector in the actual configuration
\mathbf{X}	[m]	position vector in the reference configuration
χ_α		function of placements
$(\dots)'$		material time derivative
\mathbf{u}_α	[m]	displacement of the constituent α
\mathbf{v}_α	[m/s]	velocity of the constituent α
\mathbf{a}_α	[m/s ²]	acceleration of the constituent α
\mathbf{w}	[m/s]	seepage velocity between the phases
\mathbf{w}_F	[m/s]	filter velocity
$\tilde{\mathbf{u}}_\alpha$	[m]	displacements in the <i>Fourier</i> -domain
$\tilde{\mathbf{v}}_\alpha$	[m/s]	velocities in the <i>Fourier</i> -domain
g	[m/s ²]	gravitational acceleration
Φ_α	[m ²]	scalar potential for the displacement field
Ψ_α	[m ²]	vector potential for the displacement field

$\hat{\Phi}_\alpha$	[m ²]	scalar potential in the <i>Fourier</i> -domain
$\hat{\Psi}_\alpha$	[m ²]	vector potential in the <i>Fourier</i> -domain
k_x, k_y	[rad/m]	wavenumbers
k_{11}, k_{12}	[rad/m]	wavenumbers of the compressional waves
k_2	[rad/m]	wavenumber of the shear wave
\mathbf{F}_α	[–]	deformation gradient
\mathbf{H}_α	[–]	displacement gradient
\mathbf{C}_α	[m]	right <i>Cauchy-Green</i> deformation tensor
\mathbf{B}_α	[m]	left <i>Cauchy-Green</i> deformation tensor
\mathbf{E}_α	[–]	<i>Green</i> strain tensor
\mathbf{A}_α	[–]	<i>Almansi</i> strain tensor
e_s	[–]	volumetric strain of the solid component
e_{sR}	[–]	part of e_s belonging to the real material
e_{sN}	[–]	part of e_s belonging to the change of pores
\mathbf{T}_α	[N/m ²]	<i>Piola-Kirchhoff</i> stress tensor of the constituent α
\mathbf{T}_α^E	[N/m ²]	tensor of effective stresses of the constituent α
\mathbf{T}_S^D	[N/m ²]	deviatoric part of the stress tensor
$\hat{\mathbf{T}}_\alpha$	[N/m ²]	<i>Piola-Kirchhoff</i> stress tensor in the <i>Fourier</i> -domain
$\sigma_\alpha, \tau_\alpha$	[N/m ²]	physical normal and shear stresses
$\hat{\sigma}_\alpha, \hat{\tau}_\alpha$	[N/m ²]	physical stresses in the <i>Fourier</i> -domain
$\tilde{\lambda}_S$	[N/m ²]	1 st macroscopic <i>Lamé</i> constant
μ_S	[N/m ²]	2 nd macroscopic <i>Lamé</i> constant
$\tilde{\lambda}_S^c, \mu_S^c$	[N/m ²]	complex macroscopic <i>Lamé</i> constants
p	[N/m ²]	pore pressure
\hat{p}	[N/m ²]	pore pressure in the <i>Fourier</i> -domain
p_α^E	[N/m ²]	effective stresses of the constituent α
p_α	[N/m ²]	partial hydrostatic stresses of the constituent α
K_{SR}	[N/m ²]	compression modulus of the real solid material
K_{SN}	[N/m ²]	compression modulus of the solid skeleton
\hat{p}_α^E	[N/m ³]	interaction forces between the constituents
\mathbf{S}_G	[Ns/m ⁴]	permeability tensor
γ_{GR}	[N/m ³]	effective fluid weight
k^G	[m/s]	<i>Darcy</i> flow coefficient
K^G	[m ⁴ /(Ns)]	specific permeability/conventional saturated permeability

K^S	$[\text{m}^2]$	intrinsic permeability
η_G	$[\text{Ns}/\text{m}^2]$	partial dynamic fluid viscosity
η_{G^R}	$[\text{Ns}/\text{m}^2]$	effective dynamic fluid viscosity
Ξ	$[\text{Ns}/\text{m}^4]$	specific flow resistance ($1 \frac{\text{Rayl}}{\text{cm}} = 10^3 \frac{\text{Ns}}{\text{m}^4}$)
θ	$[\text{K}]$	temperature
R	$[\text{J}/(\text{kg K})]$	specific gas constant ($R = 287.058$ for dry air)

Theory of Elasticity

ρ_H	$[\text{kg}/\text{m}^3]$	density of the elastic material
\mathbf{u}_H	$[\text{m}]$	displacement vector for the elastic material
\mathbf{v}_H	$[\text{m}/\text{s}]$	velocity
\mathbf{a}_H	$[\text{m}/\text{s}^2]$	acceleration
$\hat{\mathbf{u}}_H$	$[\text{m}]$	displacements in the <i>Fourier</i> -domain
$\hat{\mathbf{v}}_H$	$[\text{m}/\text{s}]$	velocities in the <i>Fourier</i> -domain
Φ_H	$[\text{m}^2]$	scalar potential for the displacement field
Ψ_H	$[\text{m}^2]$	vector potential for the displacement field
$\hat{\Phi}_H$	$[\text{m}^2]$	scalar potential in the <i>Fourier</i> -domain
$\hat{\Psi}_H$	$[\text{m}^2]$	vector potential in the <i>Fourier</i> -domain
k_x, k_y	$[\text{rad}/\text{m}]$	wavenumbers
k_3, k_4	$[\text{rad}/\text{m}]$	wavenumbers of the compressional and the shear wave
k_r	$[\text{rad}/\text{m}]$	circular wavenumber
\mathbf{E}_H	$[-]$	<i>Green</i> strain tensor
\mathbf{T}_H	$[\text{N}/\text{m}^2]$	<i>Piola-Kirchhoff</i> stress tensor for the elastic material
$\hat{\mathbf{T}}_H$	$[\text{N}/\text{m}^2]$	<i>Piola-Kirchhoff</i> stress tensor in the <i>Fourier</i> -domain
σ_H, τ_H	$[\text{N}/\text{m}^2]$	physical normal and shear stresses
$\hat{\sigma}_H, \hat{\tau}_H$	$[\text{N}/\text{m}^2]$	physical stresses in the <i>Fourier</i> -domain
λ_H	$[\text{N}/\text{m}^2]$	1 st <i>Lamé</i> constant of the elastic material
μ_H	$[\text{N}/\text{m}^2]$	2 nd <i>Lamé</i> constant of the elastic material

Acronyms

BEM	Boundary Element Method
BT	Biot's Theory
CMS	Component Mode Synthesis
EFEM	Energy Finite Element Method
FEM	Finite Element Method
FSI	Fluid-Structure Interaction
FRF	Frequency Response Function
HBEM	Hybrid Boundary Element Method
IRF	Impulse Response Function
ITM	Integral Transform Method
SFEM	Spectral Finite Element Method
SM	Spectral Method
TPM	Theory of Porous Media