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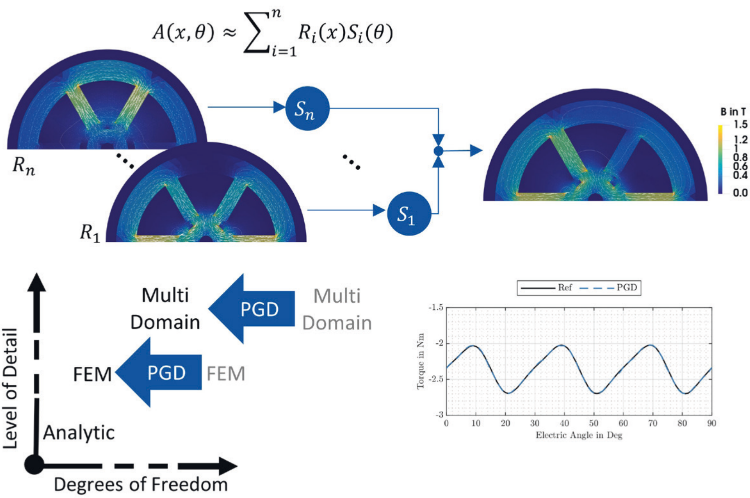
Model Order Reduction: Proper Generalized Decomposition in the Context of Non-linearity and Motion in Electrical Machines



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Band 57

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der Rheinisch-Westfälischen Technischen Hochschule Aachen
zur Erlangung des akademischen Grades eines Doktors der
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To my beloved wife Corinna.
Thank you for always having my back.

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Abstract

Motivation, Goal and Task of the Dissertation

In the design stage of electrical machines, the Finite Element Analysis is one of the most widely used numerical simulation tools to analyse electromagnetic fields and the machines' behaviour. It is able to compute complicated geometries in two- and three-dimensions with a good accuracy. However, this numerical approach may involve a huge number of unknowns, which have to be determined. To give general answers to physical and technical relevant questions, several problem classes can be distinguished. Basically, there are transient processes resulting in eddy currents, which have to be resolved in space and time. The non-linearity of the flux-guiding ferromagnetic material has to be considered and the relative motion between static and moving parts of the machine is crucial. Here, parameters vary depending on time or space. The degrees of freedom associated with the simulations, particularly if many operating points or machine design parameter combinations have to be studied, increases significantly.

To reduce the degrees of freedom of the set model, model order reduction techniques can be applied. Most of the reduction approaches are limited to linear problem formulations. This means that the achieved reduction of degrees of freedom comes at the cost of a decreased accuracy, which is unwanted. This is due to the fact that the model order reduction approach requires the underlying problem to be separable. This is not given if a problem class with non-linear material characteristics or relative motion is involved. The non-linear saturation depends on the operating point and is in general non-separable. Non-linear iteration schemes are obligatory to resolve the material behaviour, which can introduce numerical instabilities and increase the computational effort of the reduced model significantly. If the number of degrees of freedom or the connection of these changes, due to geometrical adjustments or relative motion, the underlying system of equations varies in size and sparsity pattern, which interferes with the required separation property.

A promising model order reduction technique to enable the computation of electromagnetic fields is the Proper Generalized Decomposition. It can be adapted and extended with dedicated numerical techniques to cancel these limitations to be employable in the simulation of electrical machines. To cope with the computational load of the evaluation of the non-linear material saturation, the Proper Generalized Decomposition is combined with the Discrete Empirical Interpolation Method. To lift the limitation of the Proper Generalized Decomposition to conformal meshes, techniques such as inhomogeneous Dirichlet constraints and Lagrange Multipliers are employed. The Proper Generalized Decomposition is adapted and extended with these methods to consider the different requirements of the numerical field model to reduce the degrees of freedom as well as the computational effort while keeping a technical relevant accuracy.

Scientific Contributions

The field of model order reduction techniques states an area of conflict between decreasing the degrees of freedom and computational effort while keeping a desired technical accuracy of the solutions. In order to obtain a reduced representation, the Proper Generalized Decomposition is employed. It is based on an enrichment process, which does not rely on the knowledge of previously computed solutions and enriches information until the defined accuracy is achieved. In this work, the Proper Generalized Decomposition is implemented with particular focus on the simulation of low frequent electromagnetic fields as can be found in electrical machines. A combination of the Proper Generalized Decomposition with three-dimensional magnetodynamic formulations is constructed and studied on a benchmark problem in terms of accuracy of local and global quantities and reduction of degrees of freedom. To enable the parametric study of electrical machines, the Proper Generalized Decomposition is extended to field excitation related parameters, such as the current angle and amplitude in the rotating system of the machine's rotor, the direct current of an electrically excited synchronous machine and the magnetic remanence flux density of hard magnetic material. An approach is developed to include the relative motion of the rotor as a parameter into the Proper Generalized Decomposition by using the Sliding Interface Technique, which is based on Lagrange Multipliers, to lift the restriction to conformal meshes. The derived algorithms are applied to the example simulation of a synchronous generator coupled to the grid. To be able to assess the maximum possible factor of reduction in terms of degrees of freedom and computational effort, the standard Finite Element analysis is compared to the Proper Generalized Decomposition in detail.

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List of Abbreviations

ADS	Alternative Direction Scheme
DEIM	Discrete Empirical Interpolation Method
DOF	degrees of freedom
FEM	finite element method
GP	Gauß points
IEM	Institute of Electrical Machines
iMOOSE/pyMOOSE	innovative Modern Object Oriented Solver Environment
MOR	model order reduction
NoS	number of snapshots
PGD	Proper Generalized Decomposition
PMSM	permanent magnet synchronous machine
POD	Proper Orthogonal Decomposition
ROM	reduced order model
RSST	Rotational Single Sheet Tester
RWTH	Rheinisch-Westfälische Technische Hochschule
SVD	singular value decomposition
TEAM	Testing Electromagnetic Analysis Methods