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Robust Design Optimization Based on Metamodeling Techniques

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Abstract. In this thesis, the idea of robust design optimization is adopted to improve the quality of a product or process by minimizing the deteriorating effects of variable or not exactly quantifiable parameters. Robustness can be achieved via different formulations, which are compiled and discussed in the present work. All of these formulations have in common that they require many function evaluations throughout the optimization process. Especially in the growing field of computational engineering, the governing equations are typically not explicit functions but rather a nonlinear system of equations – for instance, derived from a nonlinear finite element discretization. In this case, even pointwise solutions can be quite expensive to evaluate. To reduce the tremendous numerical effort related to the described robustness analyses, metamodeling techniques are used replacing the actual numerical analysis codes by a simpler formulation. In this thesis, a method is proposed to sequentially augment the significance of metamodels for robust design optimization through additional sampling at infill points. As a result, a robust design optimization can be applied efficiently to engineering tasks that involve complex computer simulations. Even though the suggested approach is applicable to many engineering disciplines, the present work is focused on problems in the field of structural mechanics.

Robust Design Optimierung mit Hilfe von Metamodellierungstechniken

Zusammenfassung. In dieser Arbeit wird die Idee der Robust-Design-Optimierung aufgegriffen, deren Ziel es ist, die Qualität eines Produktes oder Prozesses dadurch zu verbessern, dass die störenden Auswirkungen von variablen oder nicht genau quantifizierbaren Parametern reduziert werden. Robustheit kann durch verschiedene Formulierungen erreicht werden, die in dieser Arbeit zusammengetragen und diskutiert werden. Alle diese Ansätze haben gemein, dass sie im Laufe der Optimierung viele Auswertungen der Systemgleichungen erfordern. Insbesondere für das aufstrebende Gebiet des *Computational Engineering* ist es typisch, dass das betrachtete System nicht durch geschlossen darstellbare Formeln beschrieben wird, sondern vielmehr durch ein nichtlineares Gleichungssystem, wie es z.B. aus einer nichtlinearen Finite-Elemente-Diskretisierung entsteht. In einem solchen Fall sind meist selbst punktweise Auswertungen der Systemgleichungen recht zeitaufwändig und damit teuer. Um den numerischen Aufwand von Robustheitsanalysen dennoch überschaubar zu halten, werden hier Metamodelltechniken verwendet, mit deren Hilfe das teure Originalproblem durch eine simplere Formulierung ersetzt wird. In dieser Arbeit wird eine Methode vorgeschlagen, mit der die Aussagekraft von Metamodellen in Bezug auf die Robustheit des Systems durch Hinzunahme von neuen Stützstellen sequentiell verbessert wird. Auf diese Weise kann eine Robust-Design-Optimierung auch auf Ingenieurprobleme angewendet werden, die durch aufwändige Computersimulationen beschrieben werden. Das hier vorgestellte Verfahren kann in vielen Feldern des Ingenieurwesens eingesetzt werden, im Fokus der vorliegenden Arbeit sind jedoch strukturmechanische Problemstellungen.

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