Additive Representations of Integers in Number Theory

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Preface

The text fills a last greater gap in the sequence of the author's basic studies in number theory. In this discipline, the representation of numbers by integers is a central effort. The theory of primes and factorisation shows in a first step representations of natural numbers by products. In a second step, integers with or without special characteristics can be represented by integer sum functions of one or more natural numbers, which may be primes or composite numbers. In only few cases, there can be direct methods for finding solutions to equations or tests for the possibility of solutions. Within the latter, congruences of numbers and functions can be regarded.

While the basic theory of primes and factorisation can be found in [Kr11], in this text the representation of integers by sums is discussed. Here, you can find in seven chapters binary quadratic forms of two variables, general quadratic forms and algorithms, remarks about general forms, theoretical and practical representations of natural numbers, some useful congruences, and Goldbach's Conjecture. After the text, a choice of literature, collected corrections to former books of the author, and a complete index are given.

I would appreciate discussions, remarks, and hints if there are mistakes.

Leimen, in December 2009 Uwe Kraeft

http://www.kannitverstan.net/ shows contents and recent corrections of this sequence of texts

Choice of symbols

$\Rightarrow, \Leftarrow, \Leftrightarrow$	by this follows (in the given directions)
$\in$	is element of (is contained in)
$\mathbb{N}$	natural numbers 1, 2, 3, ...
$\mathbb{N}^0$	natural numbers with zero 0, 1, 2, 3, ...
P	primes 2, 3, 5, 7, 11, ...
p_I	prime I (4m+1), $m \in \mathbb{N}$
p_{II}	prime II (4n-1), $n \in \mathbb{N}$
$\mathbb{Z}$	integers ..., -3, -2, -1, 0, 1, 2, 3, ...
A={a,b,c}	an example of a set A with elements a, b, and c
a,b,α,β, ...	in this text, in most cases integers or natural numbers
$\underline{x}$	$=(x_1,x_2,\ldots,x_n)$ vector
$\lvert a \rvert$	absolute (positive) value of a number
$\lvert A \rvert$	=det(A) determinant of a matrix A
(x,y)=δ, d	gcd of x and y is δ or d (also d for discriminant)
F	form
f	polynomial or general function
(a,b,c)	$= F(x,y) = ax^2 + bxy + cy^2$ binary quadratic form
$\equiv$	$a \equiv b \pmod{c} \Leftrightarrow a \equiv b_c \Leftrightarrow (a-b)/c \in \mathbb{Z}$ for $a,b \in \mathbb{Z}$, $c \in \mathbb{N}$
$\left(\frac{a}{p}\right)$	$=(a'/p)=b \Leftrightarrow a^{(p-1)/2} \equiv b \pmod{p}$ for $(p>2) \in P$
	and normally (a,p)=1 (Legendre's symbol)
#(...)	number of ...

Other special symbols are explained in the text.

gcd	greatest common divisor
f.e.	for example (e.g.)

The order of this sequence of texts on number theory is twofold. The order following the date of printing is given at the end of this book. Another grouping is got by the colours of the covers after disciplines as follows:

arithmetic number theory:	light blue
sequences and series:	dark green
Diophantine Equations:	orange
algebraic number theory:	dark red
topological number theory:	purple
analytic number theory:	dark blue
statistical number theory:	light green
special numbers:	dark yellow
textbooks:	light yellow

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