### Fachgebiet Methoden der Signalverarbeitung Technische Universität München

# Minimax Robustness in Signal Processing for Communications

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## Berichte aus der Kommunikationstechnik

## **Muhammad Danish Nisar**

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#### **Synopsis:**

From a signal processing for communications perspective, three fundamental transceiver design components are the channel precoder, the channel estimator, and the channel equalizer. The optimal design of these blocks is typically formulated as an optimization problem with a certain objective function, and a given constraint set. However, besides the objective function and the constraint set, their optimal design crucially depends upon the adopted system model and the assumed system state. While, optimization under a perfect knowledge of these underlying parameters (system model and state) is relatively straight forward and well explored, the optimization under their imperfect (partial or uncertain) knowledge is more involved and cumbersome. Intuitively, the central question that arises here is: should we fully trust the available imperfect knowledge of the underlying parameters, should we just ignore it, or should we go for an "intermediate" approach?

In this dissertation, we explore the concept of minimax robustness, that falls under the generic framework of deterministic optimization under uncertainty, for the aforementioned design problems under an imperfect knowledge of the underlying parameters. First, we present the design of a minimax robust precoder under an uncertainty knowledge of the transmission channel, pursued via minimax optimization with a novel uncertainty class that aims to reduce the conservativeness of the existing robust precoder designs. Second, we discuss the design of a channel equalizer that is robust against uncertainty in the knowledge of the transmission channel as well as the interference and noise correlations, and simultaneously is able to exploit the apriori information about the unknown transmit symbols. Third, we investigate the problem of pilot assisted channel estimation, and present the design of a novel minimax robust channel estimator, once only a coarse knowledge about channel correlations is available.

Thus, this thesis deals with three crucial transceiver design problems from a signal processing for communications perspective, and attempts to answer the fundamental question of how to handle the presence of uncertainty about the design parameters in the respective optimization problem formulations.

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Munich, March 2011,

Muhammad Danish Nisar.

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