

Investigation of Trailing-Edge Blowing on Airfoils for Turbomachinery Broadband Noise Reduction

Dissertation
zur Erlangung des akademischen Grades
DOKTOR-INGENIEUR

vorgelegt von
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eingereicht dem
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Tag der mündlichen Prüfung:
17. Juni 2011

Berichte aus der Strömungstechnik

Julian Winkler

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on Airfoils for Turbomachinery
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Shaker Verlag
Aachen 2011

Bibliographic information published by the Deutsche Nationalbibliothek

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Zugl.: Siegen, Univ., Diss., 2011

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Printed in Germany.

ISBN 978-3-8440-0350-5

ISSN 0945-2230

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen

Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9

Internet: www.shaker.de • e-mail: info@shaker.de

Kurzfassung

Aerodynamisch generierter Schall stellt in vielen Turbomaschinenanwendungen, wie in Flugzeugtriebwerken, Windturbinen, Hubschraubern, sowie in Kühlungsgebläsen vieler industrieller Prozesse, eine Hauptlärmquelle dar. Die vorliegende Studie befasst sich mit dem Hinterkantenausblasen als Möglichkeit zur Breitbandschallreduktion in Turbomaschinen. Jenes Ausblasen wurde in der Vergangenheit hauptsächlich für die Schallreduktion im Triebwerkssfan erprobt, ist aber prinzipiell auch denkbar für andere Turbomaschinenanwendungen. Die grundlegende Idee ist es einen sekundären Luftstrom durch die Hinterkante der Rotorblätter zu leiten, mit welchem die Rotornachlaufdelle aufgefüllt und die Rotornachlaufturbulenz reduziert wird. Dadurch ist es möglich den Schall zu reduzieren, der aufgrund der Interaktion des Rotornachlaufs mit der Statorblätter entsteht (Rotor-Stator-Interaktionsschall). Es konnte bisher nachgewiesen werden, dass durch Ausblasen sowohl der tonale als auch der breitbandige Anteil des Rotor-Stator-Interaktionsschalls reduziert werden kann. Bezuglich des Breitbandschalls fehlt jedoch bisher eine detaillierte Analyse aller relevanten Schallentstehungsmechanismen in Verbindung mit dem Ausblasen.

Hinterkantenausblasen zur Breitbandschallreduktion wird in dieser Studie für niedrige MACH-Zahlen und hohe REYNOLDS-Zahlen am Einzeltragflügel und an zwei Tragflügeln in Tandemverbund untersucht. Dazu werden unterschiedliche Ausblageometrien betrachtet. Experimentelle Untersuchungen dazu werden im aeroakustischen Windkanal durchgeführt und beinhalten Messungen des Schallfeldes, des instationären Tragflügeldruckfeldes und des turbulenten Nachlaufs unter Einfluss des Hinterkantenausblasens. Zusätzlich wird für eine detaillierte Analyse der relevanten Schallentstehungsmechanismen die inkompressible Strömung um den Einzeltragflügel für verschiedene Ausblageometrien mittels Grobstruktursimulationen berechnet. Der abgestrahlte Schall wird dann für eine bestimmte Ausblasgeometrie auf Basis verschiedener analytischer Modelle und einem numerischen Ansatz berechnet und mit Messungen verglichen.

Es wird gezeigt, dass Hinterkantenausblasen zwei Schallentstehungsmechanismen gegenläufig beeinflusst. Während die Nachlaufturbulenz in Amplitude als auch Raum-Zeit-Struktur reduziert werden kann und damit auch der Nachlauf-Tragflügel Interaktionsschall (welcher dem Rotor-Stator-Interaktionsschall der Turbomaschine entspricht) im Frequenzbereich unterhalb von 2 kHz, erhöht sich das Eigengeräusch des Ausblasflügels für Frequenzen oberhalb von 2 kHz. Diese Eigengeräuscherhöhung entsteht vorwiegend durch die Interaktion des Ausblasstrahls mit der Flügelhinterkante und vermindert deutlich das Gesamtschallreduktionspotential des Hinterkantenausblasens. Es wird gezeigt, dass beim Ausblasen durch einen breiten Schlitz nahe der Hinterkante dennoch eine Reduktion von maximal 2.9 dB im Summenpegel möglich ist. Wird dieser Schlitz in viele Einzelkanäle unterteilt, so erhöht sich die maximale Schallreduktion auf einen Wert von 4.1 dB, aufgrund der besseren Durchmischung des austretenden Strahls mit dem externen Stromfeld. Die genannten Schallreduktionswerte werden bei teilweisem, nicht vollständigem Auffüllen der Rotornachlaufdelle erreicht. Für solch ein partielles Auffüllen wird weiterhin tonaler Schall in einer Turbomaschine erwartet. Wird die Nachlaufdelle im Sinne eines impulslosen Nachlaufs optimal aufgefüllt, so reduziert sich das Gesamtschallreduktionspotential aufgrund des Ausblaseigengeräusches des Tragflügels. In dem Fall können additive passive Maßnahmen erforderlich sein um das hochfrequente Eigengeräusch des Ausblasflügels zu reduzieren. Es wird gezeigt, dass eine poröse Hinterkante zu diesem Zweck geeignet ist. Die untersuchten gezackten Hinterkanten hingegen stellen sich im vorliegenden Fall als weitgehend akustisch nachteilig heraus.

Abstract

Aerodynamically generated sound is the major cause for large noise emission from turbomachinery including turbofan engines, wind turbines, helicopters, and cooling fans in a large variety of industrial applications. The following study deals with a method for turbomachinery noise reduction called trailing-edge blowing. This method is, in principle, applicable to any turbomachinery application but has so far been intended almost exclusively for noise reduction in the fan stage of modern turbofan engines. This method involves the addition of a secondary air flow through the trailing edge of a rotor, with which the rotor wake momentum deficit is filled and its turbulence is reduced. This energized wake produces less rotor-stator interaction noise when it impinges on a downstream stator vane. While this method has proven to be effective for tonal noise reduction, investigations regarding the broadband noise reduction potential have largely lacked a detailed analysis of all relevant noise mechanisms that are affected by this method.

Trailing-edge blowing for broadband noise reduction will be studied here on single airfoils and on two airfoils in tandem at low MACH numbers and high REYNOLDS numbers. The investigations will be conducted on different trailing edge blowing designs. Experiments will be conducted in an aeroacoustic wind tunnel and include measurements of the acoustic far field, the unsteady pressure sources on the airfoils, and the turbulent wake manipulated by blowing. In addition, several incompressible large-eddy simulations will be performed for a detailed analysis and assessment of the turbulent noise sources. The acoustic far-field will be predicted from these simulations using different analytical models and a computational aeroacoustic approach.

It will be shown that trailing-edge blowing affects two competing noise mechanisms. The wake turbulence is successfully reduced by trailing-edge blowing and a noise reduction for frequencies below 2 kHz can be observed when this modified wake impinges on a downstream airfoil (wake-airfoil interaction noise). However, the blowing jet itself produces high-frequency noise above 2 kHz as it interacts with the trailing edge of the blowing airfoil (blowing self-noise). This additional self-noise diminishes the noise reduction through wake manipulation. The overall integrated noise reduction potential will be shown to be of the order of 2.9 dB for an airfoil equipped with a single wide blowing slot. Improvements can be made by segmenting the spanwise slot into an array of discrete channels. This increases the blowing velocity while reducing the required mass-flow rate to add the same blowing momentum. The overall noise reduction is enhanced to 4.1 dB due to an improved mixing of the blowing jet with the external airfoil flow. These reduction levels occur for partial wake-filling, for which the tonal noise—as a result of the mean wake velocity deficit—would not be fully eliminated in a turbomachine. For complete wake-filling conditions the broadband noise reduction can be considerably lower, depending on the blowing geometry. In that case, additional complementary techniques may become necessary to passively reduce the high-frequency blowing self-noise. It will be shown that a porous edge can help to reduce the acoustic efficiency of the blowing jet interaction with the trailing edge and thereby improve the overall possible noise reduction levels from trailing-edge blowing. Conversely, serrated edges will largely yield negative acoustic effects for the configurations investigated in this study.

Acknowledgments

First and foremost I express my sincerest gratitude to my Ph.D. advisor Prof. Dr.-Ing. Thomas Carolus, who established a work environment that nurtures team work, but also a work ethic that produces curiosity, creativity, and freedom. I have truly enjoyed the highest degree of freedom to conduct my research and to follow paths which we did not know where they may lead us. In the same breath, I would like to acknowledge my appreciation for the continuous support I have received from Prof. Stéphane Moreau, Ph.D., of Université de Sherbrooke. We met in the summer of 2007 for the first time, and right from the beginning he offered to support me in my research efforts. Ever since the Summer Program 2008 at Stanford University, I was able to perform computations with licenses and computer time provided by him under his sponsorship and was able to continuously discuss the numerical and analytical results I had obtained.

I am extremely thankful to my present and past colleagues and fellow students in Siegen, who helped to give me a jolly time at university in the laboratory, the office, and the coffee breaks. In particular, I am thankful to Dr.-Ing. Hauke Reese; Dr.-Ing. Daniel Wolfram; Dipl.-Ing. Ralf Starzmann; Dipl.-Ing. Michael Kohlhaas; Konrad Bamberger, M. Sc.; our wonderful secretary Christine Krause; and the irreplaceable Dipl.-Ing. Bernd Homrighausen—our work-horse when it comes to CAD, experimental setups and great ideas. The PIV measurements presented in this work were carried out jointly with my former colleague Dipl.-Ing. Jörg Scheuerlein, Ph.D. student with Prof. Dr. rer. nat. Friedrich Dinkelacker, now at Leibniz Universität Hannover. My thanks go to both of them for their help, equipment, curiosity and interest in this topic. Due to space and time constraints, only a very limited part of the huge amount of data we collected found its way into this dissertation.

I gratefully acknowledge the support from our workshop in Siegen, operated by F. Stahl and W. Schröder, and their fine craftsmen; especially Ernst Bender, who performed the extremely skillful and very delicate work on all the airfoil models.

The experimental study on airfoil noise generation with the acoustic polars presented in Chapter 4 of this thesis, is based on the report “Experimentelle Ermittlung der Auftriebs- und Schallpolaren von Ventilatorschaufelprofilen im Windkanal” and was fully sponsored by the Forschungsgemeinschaft für Luft- und Trocknungstechnik (FLT) under grant L222. The computations presented in this work were performed on the RUBENS cluster at Universität Siegen and the MAMMOUTH cluster of the Réseau Québécois de Calcul Haute Performance (RQCHP) located at Université de Sherbrooke in Canada. Initial computations were performed on the JHF cluster at the Center for Turbulence Research (CTR) at Stanford University during the Summer Program 2008. The attendance to this program was financially supported by the Deutsche Forschungsgemeinschaft (DFG, German Research Foundation) under grant WI3501/1-1. The research visit to Sherbrooke in the summer of 2010 was financed by the Deutscher Akademischer Austauschdienst (DAAD, German Academic Exchange Service) under grant D/10/45322.

Last but not least, I would like to thank my parents Gerda and Richard Winkler and my wife Abigail for their unconditional love and support.

*To my parents,
Donna and Ritchi,
and to my wife,
Abby*

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Nomenclature

Latin Symbols

a	grid bar width	[m]
a_-, a_+	lower, upper bound of value	[units(a_{\pm})]
b	constant in CORCOS's model	[\cdot]
C	contraction ratio of wind-tunnel nozzle	[\cdot]
C_d	dynamic constant in the SMAGORINSKY model	[\cdot]
C_{ij}	cross-stress tensor	[$(\text{m}/\text{s})^2$]
C_s	constant in the classical SMAGORINSKY model	[\cdot]
c	airfoil chord length	[m]
c_d	drag coefficient	[\cdot]
c_f	skin-friction coefficient	[\cdot]
c_l	lift coefficient	[\cdot]
c_p	pressure coefficient	[\cdot]
c_0	speed of sound	[m/s]
c_μ	momentum coefficient	[\cdot]
$c_{\mu,\text{net}}$	net blowing momentum coefficient	[\cdot]
$\bar{c}_{\mu,\text{net}}$	referenced net blowing momentum coefficient	[\cdot]
d	nozzle width	[m]
$E(y)$	expanded uncertainty of y	[units(y)]
E_i	1-D energy spectrum of the i^{th} velocity component	[$(\text{m}^2/\text{s}^2)/\text{Hz}$]
E_{kin}	turbulent kinetic energy	[$(\text{m}/\text{s})^2$]
E^*, \mathcal{F}	combinations of FRESNEL integrals	[\cdot]
\mathcal{E}_{ij}	error associated with the dynamic SMAGORINSKY model	[$(\text{m}/\text{s})^2$]
erf	error function	[\cdot]
$F(\dots)$	filter kernel	[\cdot]
F_i	body force per unit volume in i direction	[N/m ³]
$\mathcal{F}_1, \mathcal{F}_2$	FRESNEL integrals	[\cdot]
f_i	body force per unit mass in i direction	[N/kg]
f, f_s	frequency, sampling frequency	[Hz]
G	GREEN's function	[1/m]
G_{xx}	onesided PSD of reference microphone signal	[Pa ² /Hz]
G_{yy}	onesided PSD of pressure sensor signal	[V ² /Hz]
G_{xy}	onesided CSD between pressure sensor and reference microphone signals	[PaV/Hz]
G_{12}	onesided CSD between microphone signals M1, M2	[Pa ² /Hz]

g_1, g_2	transfer functions in AMIET's LEN theory	[\cdot]
$H_{\mathcal{X}\mathcal{Y}}$	transfer function between pressure sensor and reference microphone signals	[V/Pa]
H_I, H_{LE}, H_{TE}	transfer functions in AMIET's TEN theory	[\cdot]
$H_0^{(1)}, H_1^{(1)}$	HANKEL functions of the first kind	[\cdot]
$h_{airfoil}$	maximum thickness of airfoil	[m]
h_{BLT}	boundary layer trip thickness	[m]
h_{serr}	trailing edge serration height	[m]
h_{slot}	blowing slot height	[m]
h_{TE}	trailing-edge thickness	[m]
$\Im(arg)$	imaginary part of arg	[units(arg)]
\mathcal{I}	radiation integral in AMIET's TEN model	[\cdot]
J_0, J_1	BESSEL functions of the first kind	[\cdot]
k	acoustic wavenumber	[1/m]
k_c	convective wavenumber	[1/m]
k_{cov}	coverage factor	[\cdot]
k_e	wavenumber in the VON KÁRMÁN model	[1/m]
k_0, k_1, k_3	freestream, streamwise, spanwise wavenumber	[1/m]
L	airfoil span	[m]
L_{ij}	LEONARD stress tensor	[$(m/s)^2$]
L_{LES}	simulated airfoil span	[m]
L_{slot}	slot width	[m]
L_x, L_y, L_z	computational grid point count	[\cdot]
L_{Ei}	1-D energy spectrum level of velocity component i	[dB]
L_{Spp}	acoustic pressure PSD level	[dB]
$L_{\Phi pp}$	surface pressure PSD level	[dB]
\mathcal{L}	radiation integral in AMIET's LEN model	[\cdot]
\mathcal{L}_{ij}^{SGS}	GERMANO identity	[$(m/s)^2$]
l_m	mixing lengthscale	[m]
l_3	spanwise correlation length	[m]
M	MACH number	[\cdot]
M	mesh size	[m]
M_{ij}^{SGS}	scaled composite rate-of-strain tensor	[$(m/s)^2$]
\dot{m}	mass-flow rate	[kg/s]
N	total number of a quantity or variable	[\cdot]
n_d	number of ensemble records	[\cdot]
n_i, n_j	normal vector	[\cdot]
n_s	number of samples	[\cdot]
$OSPL$	overall sound pressure level	[dB]

$P_{p,\text{total}}$	total power of pressure fluctuations	[Pa ²]
p_{ij}	compressive stress tensor	[Pa]
p'	hydrodynamic/acoustic pressure fluctuations	[Pa]
p'_I, p'_S	incident, scattered pressure fluctuations	[Pa]
p_{dyn}	dynamic pressure	[Pa]
p_{ref}	reference pressure ($= 2 \times 10^{-5}$ Pa)	[Pa]
p_{stat}	static pressure	[Pa]
pdf	probability density function	[\cdot]
Q	second invariant of the velocity gradient tensor	[1/s ²]
R	distance between source and observer	[m]
R'	distance between image source and observer	[m]
R_0	universal gas constant of air	[J/(kgK)]
$\Re(arg)$	real part of arg	[units(arg)]
$R_{pp}(x, y, \xi, \eta, t, \tau)$	two-point, two-time pressure correlation tensor	[Pa ²]
$R_{ij}(x, y, \xi, \eta, t, \tau)$	two-point, two-time velocity correlation tensor	[(m/s) ²]
$R_{ij}(x, t)$	REYNOLDS stress (one-point, one-time velocity correlation) tensor	[(m/s) ²]
Re	REYNOLDS number	[\cdot]
S	integration surface of solid body	[m ²]
S_{FWH}	source term in FWH's theory	[m ^{7/2} /s ²]
S_0	convection-corrected far-field observer position	[m]
S_{ij}	rate-of-strain tensor	[1/s]
S_{pp}	acoustic pressure PSD	[Pa ² /Hz]
$S_{\Delta p' \Delta p'}$	PSD of unsteady pressure jump across airfoil	[Pa ² /Hz]
\bar{S}	filtered rate-of-strain invariant	[1/s]
S_{SEARS}	SEARS function	[\cdot]
S_{ij}^d	mixture of rate-of-strain tensor and rate-of-rotation tensor in the WALE model	[1/s ²]
Sr	STROUHAL number	[\cdot]
SPL	sound pressure level	[dB]
T	total record length	[s]
T_0	ambient air temperature	[K]
T_{ij}	LIGHTHILL stress tensor	[Pa]
TI_i	turbulence intensity of the i^{th} velocity component	[\cdot]
\bar{T}	temporal cut-off scale	[s]
$\mathcal{T}_{ij}^{\text{SGS}}$	subgrid stress tensor for coarser LES test filter	[(m/s) ²]
t	time	[s]
t_s	STUDENT parameter	[\cdot]
t_m	mixing timescale	[s]

u_0, u_c, u_{jet}	freestream, convection, jet velocity	[m/s]
u_n	wall-normal velocity	[m/s]
$u_R, u_{R'}$	integration limits in FWH's theory	[\cdot]
u_r, u_θ, u_z	cylindrical polar velocity components	[m/s]
u, v, w	streamwise, crosswise, and spanwise velocity components in Cartesian coordinates	[m/s]
U, W, C	circumferential, relative, and absolute velocity components in a rotating frame of reference	[m/s]
u', v', w'	velocity fluctuations	[m/s]
V	integration volume around the airfoil	[m ³]
\dot{V}	volume-flow rate	[m ³ /s]
r, θ, z	cylindrical polar coordinates of observer field point	[m], [rad], [m]
r_0, θ_0, z_0	cylindrical polar coordinates of source field point	[m], [rad], [m]
X	random input variable	[units(X)]
\mathcal{X}	FOURIER transform of signal x	[units(\mathcal{X})]
x, y, z	streamwise, crosswise, and spanwise Cartesian coordinates	[m]
x^+, y^+, z^+	streamwise, crosswise, and spanwise Cartesian coordinates in wall units	[\cdot]
x_1, x_2, x_3	streamwise, crosswise, and spanwise Cartesian coordinates of observer location	[m]
Y	random output variable	[units(Y)]
\mathcal{Y}	FOURIER transform of signal y	[units(\mathcal{Y})]
y_1, y_2, y_3	streamwise, crosswise, and spanwise Cartesian coordinates of source location	[m]
y_n	wall-normal direction	[m]
\mathcal{W}	test function in variational formulation of Lighthill's analogy	[\cdot]

Greek Symbols

α, α_{eff}	geometric, effective angle of attack	[rad]
β	compressibility parameter	[\cdot]
$\bar{\Delta}, \tilde{\Delta}$	spatial cut-off scale of LES filter, coarse LES test filter	[m]
Δf	frequency resolution	[Hz]
$\Delta p'$	unsteady pressure jump across airfoil	[Pa]
$\Delta \bar{t}$	dimensionless time-step size	[\cdot]
$\delta(\dots)$	DIRAC delta function	[\cdot]
δ_{ij}	KRONECKER delta	[\cdot]

δ^*	displacement thickness	[m]
δ^{**}	sum of displacement and momentum thicknesses	[m]
η_K	KOLMOGOROV lengthscale	[m]
$\epsilon(y), \epsilon_c(y)$	standard, combined standard uncertainty of y	[units(y)]
$\epsilon_r(y), \epsilon_{c,r}(y)$	relative, combined relative uncertainty of y	[\cdot]
Γ	gamma function	[\cdot]
γ^2	coherence function	[\cdot]
κ_0	heat capacity ratio of air at ambient conditions	[\cdot]
Λ	characteristic integral lengthscale of the flow	[m]
$\Lambda_{i,j}$	integral lengthscale of velocity component j in i direction	[m]
$\Lambda_{t,j}$	integral timescale of velocity component j	[s]
λ, λ_{gust}	acoustic wavelength, gust wavelength	[m]
μ, μ'	shear, bulk coefficient of viscosity	[kg/(ms)]
μ_{x_i}	mean value of the random variable X_i	[units(X_i)]
$\bar{\mu}$	reduced frequency associated with 2-D gusts	[\cdot]
ν	kinematic viscosity	[m ² /s]
ν_i	degree of freedom of x_i	[\cdot]
ν^{SGS}	subgrid scale eddy viscosity	[m ² /s]
Ω_{ij}	rate-of-rotation tensor	[1/s]
$\Omega_{\text{IE}}, \Omega_{\text{LH}}$	integration volumes for CAA	[\cdot]
ω	angular frequency	[rad/s]
Φ, Φ_{pp}	wall-pressure CSD, PSD	[Pa ² /Hz]
ϕ	phase of wall pressure between two locations on the airfoil	[rad]
ϕ_{xy}	phase between pressure sensor and reference microphone signals	[rad]
ϕ_{12}	phase between microphone signals M1 and M2	[rad]
φ	angle in FWH's model formulation	[rad]
Π_{pp}	wavenumber-frequency spectrum of wall pressure	[Pa ² m ² /Hz]
$\Pi_{pp,0}$	streamwise-integrated wavenumber-frequency spectrum of wall pressure	[Pa ² m/Hz]
Π_{vv}	2-D upwash-velocity wavenumber spectrum	[m ⁴ /s ²]
π_{ij}	momentum-flux tensor	[Pa]
Ψ	velocity potential	[m ² /s]
ψ	angle between observer location and trailing edge	[rad]
ψ_0	angle between source location and trailing edge	[rad]
ρ	density	[kg/m ³]
$\rho_0, \rho_{jet}, \rho_m$	density of air in freestream, jet, mixture	[kg/m ³]
ϱ	generic fluid variable	[units(ϱ)]
σ	solidity	[\cdot]

σ_{ij}	viscous stress tensor	[Pa]
$\sigma_{x_i}^2$	variance of the random variable X_i	[units(X_i^2)]
Θ_{jet}	added jet momentum displacement thickness	[m]
Θ_{wake}	wake momentum deficit (displacement thickness)	[m]
Θ_0	wake momentum deficit without blowing	[m]
τ	time separation	[s]
τ_{ij}^{SGS}	subgrid stress tensor	[$(\text{m}/\text{s})^2$]
τ_w	wall shear stress	[Pa]
ξ, η	streamwise, spanwise separation distance	[m]

Superscripts

d	deviatoric part of tensor
SGS	subgrid scale
α	parameters in coordinates rotated by α
0	ideal quiescent medium at rest

Subscripts

c	chord length
eff	effective
FS	free space
HP	half plane
l	lower
N	norm conditions
ref	reference
rms	root-mean squared
u	upper
$\Delta\dot{m}$	added mass flow through wake

Operators

$E\{\dots\}$	statistically expected value
$\langle\dots\rangle$	statistically averaged, mostly ensemble-averaged
\dots	dimensionless quantity
$\widetilde{\dots}$	filtered components in LES theory

...	filtered components in LES theory by coarse filter, otherwise “estimated value”
*	complex conjugate

Abbreviations

ATEB	advanced/alternating trailing-edge blowing
<i>arg</i>	argument
BEM	boundary-element method
BLN, BLT	boundary-layer noise, boundary-layer trip
CAA	computational aeroacoustics
CAD	computer-aided design
CFD	computational fluid dynamics
CFL	COURANT-FRIEDRICH-S-LEWY
CG	coarse grid
CSD	cross-spectral density
DES	detached-eddy simulation
DDES	delayed detached eddy simulation
DNS	direct numerical simulation
FC	finite chord
FE, FEM	finite element, finite-element method
FG	fine grid
FWH	FFOWCS WILLIAMS & HALL
IDDES	improved delayed detached eddy simulation
IE	infinite element
IGV	inlet guide vanes
LE, LEN	leading edge, leading-edge noise
LES	large-eddy simulation
LIN	laminar-instability noise
M1,M2,...	microphone 1, 2, ...
MP1,MP2,...	measurement plane 1, 2, ...
NSE	NAVIER-STOKES equations
OGV	outlet guide vanes
PIV	particle image velocimetry
PSD	power-spectral density
RANS	REYNOLDS-averaged NAVIER-STOKES
RDT	rapid-distortion theory
RMS	root-mean squared

RSI	rotor-stator interaction, also: stator-rotor interaction
SAS	scale-adaptive simulation
SN	stall noise
TE, TEB, TEN	trailing edge, trailing-edge blowing, trailing-edge noise
TS	TOLLMIEN-SCHLICHTING
URANS	unsteady REYNOLDS-averaged NAVIER-STOKES
VSN	vortex-shedding noise
WAI	wake-airfoil interaction
WALE	wall adaptive local eddy viscosity
ZG	zero grid
1-D, 2-D, 3-D	one-, two-, three-dimensional