

Conversion of representations of 3D geo-objects

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Mai Phuong Vuong

Abstract

Ever increasingly huge sets of data indexed with reference to 2d, 3d or 4d real space are being collected in all fields of science and require intelligent data models for well performing management. Thus we experience a growing interest in more efficient data models both in terms of memory and management, especially search and retrieval.

Two features involve modeling objects: geometry and topology. Our data model developed here comprises a novel representation of the topology of objects in any spatial dimension and an implicit representation of the geometry in terms of embedding.

Based on the mathematical features of our data model, we develop two algorithms to convert objects from pixel/voxel mode to boundary representation including topology. After encoding the algorithms in the development environment provided by Graphite we apply the conversion to electron back scatter diffraction (EBSD) data to present a typical application referring to geo- and material sciences.

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