

Modellierung und Regelung komplexer dynamischer Systeme

Band 17

**Thomas Wimböck**

**Controllers for Compliant Two-Handed Dexterous  
Manipulation**

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## Preface

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The present monograph was created at the Institute of Robotics and Mechatronics of the German Aerospace Center (DLR e.V.) in collaboration with the Automation and Control Institute (ACIN) of TU Vienna. This work is based on my Ph.D. thesis, which has been submitted to the Faculty of Electrical Engineering and Information Technology of TU Vienna in December 2012 and was successfully defended.

I want to give thanks to several persons who helped and supported me during my thesis. I am very grateful to Prof. Hirzinger for giving me the possibilities to investigate robotic dexterous manipulation. I want to thank Prof. Kugi for the collaboration and the valuable and precise advices.

On this occasion I want to thank Dr. Steffen Haidacher who introduced me to the field of robot hands and dexterous manipulation. Special thanks go to Dr. Christian Ott for the close collaboration and the inspiring discussions. I also want to express my gratitude to Dr. Alin Albu-Schäffer for the constant support and precious counsels. Many thanks also to Christoph Borst for mentoring my work when joining the institute and the for the work together with Justin. For my research the DLR Hand II was fundamental and I am grateful to Jörg Butterfaß who traced back every malfunction of the hands with infinite patience whenever I broke them. Thanks to all the colleagues who contributed to Justin and keep him running. Many thanks to Berthold Bäuml for working together on catching the flying ball application. I want to also thank my colleagues Maxime Chalon, Dr. Zhaopeng Chen, Alexander Dietrich, Werner Friedl, Markus Grebenstein, Martin Görner, Sami Haddadin, Jens Reinecke, Florian Schmidt, Andreas Stemmer and Klaus Strobl for the fruitful work together and the lively discussions we had. I want to further acknowledge the work of the students who worked with me. In particular I want to thank Christian Connette, Moritz Ritter and Benjamin Jahn for their contributions. Thanks to the Seventh Framework Programme (FP7) of the European Commission who supported my research in the Project THE - The Hand Embodied.

From the visits of Prof. Nenchev and Prof. Trinkle I could gain clear insights into

the field of flexible base robots and new trends in robotic grasping. Thank you very much for sharing your experience with me. Thanks to Dr. Gianluca Palli for working together on the control of variable stiffness systems for hands during his visit to DLR. I also wish to express to my gratitude to Prof. Prattichizzo and his visiting students Marco Aggravi and Gionata Salvietti.

I was sometimes asked if I could imagine to have my robot as partner. That was easy to answer because I will never exchange my great family against a robot. I want to thank them deeply for their patience and sacrifices for me and my work - especially my mum who made it possible that I could follow this path. Many thanks to my dear Hoda <3 for her support and trust.

Vienna, Dezember 2012

Thomas Wimböck

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## Abstract

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Dexterous manipulation of large classes of objects demands large capacities from robotic systems. Most important are versatility, adaptability, and sensitivity. Robots comprised of two arms with two-jaw grippers as end-effectors have been used widely to solve various tasks. Recently, systems equipped with dexterous hands were developed, which exhibit a greater adaptability and promise a wider range of applications. The increased sensitivity of the dexterous hand together with its lower joint friction compared to arm joints is expected to enhance the overall functionality of the two-handed system.

Previous works on the control of two-handed systems consisting of two arms equipped with dexterous hands was mostly restricted to the independent control of the arm and the hand subsystems. The coordination between fingers and arms has been barely studied. When studying the work related to the object level control of robotic hands it can be seen that several publications deal with the dynamics of hand object systems, but the practitioners typically use pure stiffness controllers ignoring all the dynamic effects. In order to further develop the field of dynamic dexterous manipulation in theory and practice, this thesis is concerned with concepts to model and control a robot consisting of an upper body with two arms equipped with dexterous hands. For such a two-handed robot, the equations of motion are derived and compliant grasping controllers for various tasks are proposed. In particular, the object level impedance control was studied in more detail and a novel controller - the static intrinsically passive controller (IPC) - was proposed. In the literature, a systematic comparison of such controllers, in particular with experimental validation is missing. In the experiments of this thesis it could be shown that dynamic effects indeed are *not negligible*. The developed object level impedance controller is used in the synthesis of an impedance controller for two-handed compliant manipulation, which includes the finger arm coordination. This is the first time that such an impedance control law was successfully implemented on a two-handed robotic system - the DLR robot *Justin*. In view of the development of the new DLR (integrated) Hand Arm System, the control of tendon driven systems with inherent mechanically

varying, respectively nonlinear, stiffness was investigated. In the field of control of variable stiffness actuators many approaches have been proposed in the literature in the last decade. However, only a few research groups are working on the control design for tendon-driven robotic hands with variable stiffness. In this context, this thesis contributes to the derivation and analysis of a controller based on motor side measurements with feed forward terms. The stability proof of the closed loop system extends the seminal work of Tomei (Tomei, 1991) to the case of tendon controlled robots with variable joint stiffness.

This thesis provides a large toolbox of high-level impedance behaviors for two-handed compliant manipulation. They range from reduced complexity (synergy) coordinates to the dexterous manipulation, which is the coordinated object level control using the fingertips of the hands combined with the control of the arms and the torso. These tools offer interfaces in generalized - mostly Cartesian - coordinates to specify the motion, the internal forces, and the closed loop behavior in terms of compliance and damping coefficients. The controllers devoted to the variable stiffness system allow in addition the adjustment of the mechanical joint stiffness, which can be advantageously used to optimize additional performance criteria demanded by a given task.

This toolbox is highly relevant for the development of space robotics and service robotics applications. Furthermore, robots designed as production assistants can benefit from these developments. The presented controllers form a control framework applicable also to the commercially available DLR-HIT Hand II, which is supported by the EU Project ECHORD, the DLR Dexhand, and the DLR crawler. The synergy controller serves as a versatile testbed to study reduced complexity concepts within the EU FP7 project THE - The Hand Embodied. The concepts related to the two-arm controllers can be used on two KUKA robot arms. The proposed impedance behaviors are useful to synthesize whole-body controllers.

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## Kurzzusammenfassung

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Geschickte Handhabung von sehr unterschiedlichen Objekten erfordert grundlegende Fähigkeiten von Robotersystemen. Im Wesentlichen sind es Flexibilität, Anpassungsfähigkeit und Feinfühligkeit. Roboter, ausgestattet mit zwei Armen und Zweibackengreifern an deren Handgelenken, wurden in der Vergangenheit häufig zur Durchführung verschiedener Manipulationsaufgaben eingesetzt. Erst seit kurzem wurden Robotermodelle mit geschickten Händen entwickelt, die eine stärkere Anpassungsfähigkeit mit sich bringen und eine breite Palette an Anwendungen versprechen. Man erwartet, dass die erhöhte Feinfühligkeit von geschickten Roboterhänden und deren - im Vergleich zu Armgelenken - geringe Gelenkreibung die Fähigkeiten des Gesamtsystems erhöht.

Die Mehrheit der Arbeiten zur Regelung von derartigen zweihändigen Systemen hat sich im Wesentlichen mit der unabhängigen Regelung von Armen und Händen beschäftigt. Die Koordination zwischen Finger- und Armbewegungen eines Roboters wurde bislang kaum untersucht. Bei der Regelung von Roboterhänden in Objektkoordinaten kann festgestellt werden, dass sich einige Veröffentlichungen mit der Dynamik des Hand-Objekt-Systems beschäftigten, die praktischen Arbeiten allerdings im Wesentlichen Steifigkeitsregler verwenden und die dynamischen Effekte ignorieren. Um das Forschungsfeld der geschickten, dynamischen Handhabung sowohl in der Theorie als auch in der Praxis voranzubringen, werden in dieser Doktorarbeit Konzepte zur Modellierung und Regelung eines robotischen Oberkörpers, bestehend aus einem Rumpf, zwei Armen und zwei feinfühligen Händen, vorgestellt. Für einen derartigen zweihändigen Roboter werden die Bewegungsgleichungen hergeleitet und nachgiebige Greifregler, geeignet für unterschiedliche Aufgaben, vorgeschlagen. Diese Regler basieren auf den Bewegungsgleichungen des Gesamtsystems und erlauben es, das transiente Systemverhalten gezielt zu spezifizieren. Im Speziellen wird die Impedanzregelung in Objektkoordinaten untersucht und ein neuartiger Regler, der sogennante static intrinsically passive controller (IPC), vorgeschlagen. In der Literatur wurden derartige Regler kaum miteinander verglichen, wobei vor allem experimentelle Resultate bislang fehlten. In den Experimenten in dieser Arbeit kann man deutlich erkennen, dass

die dynamischen Effekte für drehmomentengeregelte Roboterhände tatsächlich *nicht vernachlässigbar* sind. Der entwickelte Impedanzregler in Objektkoordinaten bildet die Grundlage für die Synthese eines Impedanzreglers für zweihändige nachgiebige Manipulation. Dieser beinhaltet zugleich die Koordination zwischen den Fingern und dem Arm. Es ist das erste Mal, dass ein solcher Regler erfolgreich auf einem zweihändigen Robotersystem - dem DLR Roboter *Justin* - implementiert wurde.

In Anbetracht der Entwicklung einer neuen Roboterhand des (integrierten) DLR Hand-Arm-Systems wird in dieser Arbeit die Regelung von seilzuggetriebenen Systemen mit intrinsisch veränderbarer, nichtlinearer Steifigkeit untersucht. Zur Regelung von Aktoren mit variabler Steifigkeit wurden kürzlich einige neue Ansätze vorgestellt. Jedoch gibt es nur wenige Forschungsgruppen, die sich mit dem Reglerentwurf für seilzuggetriebene Roboterhände beschäftigen. In diesem Zusammenhang leistet die vorliegende Arbeit einen Beitrag zur Entwicklung und Analyse eines Reglers basierend auf motorseitigen Messungen und einer Vorsteuerung. Der zugehörige Stabilitätsbeweis erweitert die grundlegenden Arbeiten von Tomei (Tomei, 1991) auf seilzuggetriebene Roboter mit variabler Gelenksteifigkeit.

Diese Arbeit stellt eine umfangreiche Toolbox von High-level Impedanzverhalten für zweihändige Handhabung zur Verfügung. Die Regler bieten Schnittstellen von Komplexitätsreduzierten, sogenannten Synergiekoordinaten bis hin zur zweihändigen Regelung auf Objektebene, bei der die Fingerspitzen der Hände das Objekt kontaktieren und in Kombination mit der Regelung der Arme und des Rumpfs das Objekt bewegt wird. Mit diesen Konzepten können Bewegungen, interne Kräfte und das Verhalten des geschlossenen Regelkreises mittels des gewünschten Steifigkeits- und Dämpfungsverhalten eingeprägt werden. Die Regler für die Systeme mit variabler Steifigkeit erlauben zusätzlich das Parametrieren der mechanischen Gelenksteifigkeit, die so eingestellt werden kann, dass weitere aufgabenspezifische Optimierungskriterien erfüllt werden können.

Diese Toolbox ist sehr hilfreich für die Entwicklung von Anwendungen für Service- und Weltraumroboter. Weiterhin können Roboter, die als Produktionsassistent konzipiert sind, ebenfalls davon profitieren. Die vorgestellten Regelungskonzepte stellen ein Rahmenwerk dar, das auch für die kommerziell verfügbare DLR-HIT Hand II, welche sich im EU Projekt ECHORD wiederfindet, die DLR Dexhand und für den DLR Krabbler anwendbar ist. Der Regler basierend auf Synergiekoordinaten dient als Plattform zur Entwicklung komplexitätsreduzierter Konzepte innerhalb des EU FP7 Projektes THE - The Hand Embodied. Die Strategien zur Regelung zweier Arme können direkt auf die kommerziell verfügbaren KUKA Roboterarme angewandt werden. Die vorgestellten Impedanzregelungsverfahren können weiterhin als Bausteine zur Synthese von Ganzkörperregelungen verwendet werden.

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## List of symbols

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### General symbols

$\mathbf{A}$  Matrix of velocity constraints

$\mathbf{C}_x^y(\mathbf{q}_x, \dot{\mathbf{q}}_x)$  Matrix of Coriolis and Centrifugal terms of robot system  $x$  optionally represented w.r.t. the coordinates  $y$

$\mathbf{F}_{b,n}^c$  Wrench applied to the origin of the coordinate system  $\mathcal{B}$  and the forces and torques are defined w.r.t.  $\mathcal{C}$  with the index  $n$  a placeholder for an optional description

$\mathbf{g}_x^y(\mathbf{q}_x)$  Gravity vector of robot system  $x$  optionally represented w.r.t. the coordinates  $y$

$\mathbf{H}_b$  Homogeneous transformation, which transforms vectors represented in the frame  $\mathcal{B}$  to the inertial frame

$\mathbf{H}_{sb}$  Homogenous transformation, which transforms vectors represented in the frame  $\mathcal{B}$  to the frame  $\mathcal{S}$

$\mathbf{J}^b$  Body Jacobian matrix

$\mathbf{J}_x(\eta, \epsilon)$  Representation Jacobian matrix

$\mathcal{B}$  Body frame

$\mathcal{S}$  Inertial frame

$\mathcal{T}$  Tool frame

$\mathbf{M}_x^y(\mathbf{q}_x)$  Inertia matrix for robot system  $x$  optionally represented w.r.t. the coordinates  $y$

$\boldsymbol{\omega}_{sb}^b$  Angular velocity between the frames  $\mathcal{S}, \mathcal{B}$  represented in  $\mathcal{B}$

$\mathbf{p}_{sb}^b$	Translation vector from the frame $\mathcal{S}$ to $\mathcal{B}$ w.r.t. the frame $\mathcal{B}$ . Without prefix the reference coordinate system is the inertial one
$\mathbf{q}$	joint angle position
$\mathbf{R}_{sb}$	The rotation matrix which rotates vectors represented in $\mathcal{B}$ to $\mathcal{S}$
$\boldsymbol{\tau}_{\text{ext},x}$	External torques related to robot system $x$
$\boldsymbol{\tau}_x$	Motor torque for robot system $x$
$\mathbf{V}_{sb}^b$	Body twist between the frames $\mathcal{S}, \mathcal{B}$ . If the superscript is omitted the body frame is indicated.
$\mathbf{v}_{sb}^b$	Velocity of the origin of frame $\mathcal{B}$ w.r.t. $\mathcal{S}$ , represented in $\mathcal{B}$
$\mathbf{x}_b$	Minimal local coordinates corresponding to the frame $\mathbf{H}_b$
$t$	time

### Chapters 3-6

$\bar{\mathbf{x}}$	Generalized coordinates comprised of the the local parametrization of the virtual object frame and the coordinates related to the connecting springs
$\mathcal{I}_o$	Moment of inertia of the object defined w.r.t. the center of mass of the object
$\delta_{h,i}$	Safety distance related to joint limit avoidance of joint i
$\mathbf{D}(\mathbf{q})$	Damping matrix
$\mathbf{E}(\mathbf{q}_h, \mathbf{x}_o)$	Virtual linkage w.r.t. contact frames
$\eta$	Scalar part of a quaternion number
$\mathbf{f}_\gamma$	Stacked vector of contact forces applied to the virtual object
$\mathbf{f}_{\text{tip}}^s$	Measured fingertip force based on the 3D fingertip sensor (Butterfaß et al., 2001) with respect to the inertial frame
$\mathbf{f}_c$	Stacked vector of contact forces
$\mathbf{F}_{o,\text{ext}}$	External wrench which acts on the object frame $\mathbf{H}_o$
$\mathbf{G}$	Grasp map
$\mathbf{G}_o(\mathbf{x}_o)$	Grasp map in local coordinates
$\mathbf{H}_v$	Virtual object frame

$\mathbf{H}_{\gamma_i}$	Frame at the $i^{\text{th}}$ virtual contact
$\mathbf{H}_{c_i}$	Frame of the contact point $i$
$\mathbf{H}_{ho}(\mathbf{p}_f(\mathbf{q}_h))$	Virtual object frame
$\mathbf{H}_o$	Object frame
$\mathbf{H}_{v,d}$	Desired equilibrium frame of the virtual object
$\mathbf{J}_h(\mathbf{q}_h, \mathbf{H}_o)$	Hand Jacobian matrix
$\mathbf{K}_j$	Joint stiffness matrix
$\mathbf{K}_o$	Object stiffness
$\mathbf{K}_R$	Rotational stiffness component
$\mathbf{K}_T$	Translational stiffness component
$\mathbf{K}_z$	Stiffness matrix related to synergy coordinates
$\mathbf{K}_\eta$	Stiffness matrix related to the virtual linkage
$\mathbf{K}_{\text{cart}}$	Block diagonal stiffness matrix corresponding to $\mathbf{p}_f(\mathbf{q}_h)$
$\mathbf{K}_\phi^j$	Virtual mechanical stiffness in joint coordinates
$\mathbf{K}_{hc}$	Stiffness matrix related to the connecting springs
$\mathbf{K}_{ho}$	Stiffness matrix related to the virtual object frame
$\mathbf{K}_{vo}$	Stiffness of the spatial spring attached to the virtual object
$\mathbf{p}_f(\mathbf{q}_h)$	Stacked vector of fingertip positions
$\mathbf{Q}(\mathbf{q}_h, \mathbf{x}_o)$	Grip map
$\mathbf{q}_a$	Joint angle vector containing $\mathbf{q}_h$ and $\mathbf{x}_o$
$\mathbf{q}_h$	Joint angles of the robot hand
$\mathbf{q}_t$	Torso joint angles
$\mathbf{q}_{f_i}$	Joint angles of robot finger $i$
$\mathbf{S}$	Synergy Jacobian matrix
$\mathbf{S}_\perp$	Complement to the synergy Jacobian matrix
$\boldsymbol{\tau}_S$	Measured hand joint torques

$\boldsymbol{x}_\eta$	Coordinates of the virtual linkage
$\boldsymbol{x}_{o\eta}$	Stacked vector comprised of $\boldsymbol{x}_o$ and $\boldsymbol{x}_\eta$
$\boldsymbol{x}_o$	Local coordinates of the object
$a$	Index referring to the composite hand object coordinates comprised of $\boldsymbol{q}_h$ and $\boldsymbol{x}_o$
$d_{\text{singav}}$	Damping related to joint limit avoidance
$f$	Index refers to finger
$h$	Index refers to hand
$hl$	Index refers to left hand
$hr$	Index refers to right hand
$k_{\text{singav},i}$	Stiffness related to joint limit avoidance of joint i
$l$	Index refers to left arm
$m_o$	Object mass
$n_{fn}$	Number of fingers
$ol$	Index refers to the object in the left hand
$or$	Index refers to the object in the right hand
$r$	Index refers to right arm
$t$	Index refers to torso
$u$	Index refers to upper body comprised of the torso, right and left arm
$V_a$	Scalar spring
$V_s(\mathbf{H}_1, \mathbf{H}_2, \mathcal{K})$	Spatial spring defined w.r.t. $\mathbf{H}_1$
$z$	Synergy coordinates

## Chapter 7

$\boldsymbol{f}_m$	Tendon motor forces
$\boldsymbol{f}_t(\boldsymbol{\theta}, \boldsymbol{q})$	Tendon forces
$\boldsymbol{f}_{f,h}$	Motor friction forces in tendon coordinates

$\boldsymbol{h}_\theta(\boldsymbol{\theta})$	Tendon length changes w.r.t. motor
$\boldsymbol{h}_q(\boldsymbol{q})$	Tendon length changes w.r.t. joints
$\boldsymbol{M}_h$	Effective tendon inertia matrix comprised of motor inertia
$\boldsymbol{P}(\boldsymbol{q})$	Coupling matrix
$\boldsymbol{Q}(\boldsymbol{q})$	Stacked matrix comprised of $\boldsymbol{P}^T(\boldsymbol{q})$ and $\boldsymbol{S}_t^T(\boldsymbol{q})$
$\boldsymbol{S}_t(\boldsymbol{q})$	Jacobian matrix mapping from $\boldsymbol{f}_t$ to $\boldsymbol{s}$
$\boldsymbol{s}$	Joint stiffness in vectorized form
$\boldsymbol{\theta}$	Motor positions
$m$	Number of motors
$n$	Number of joints