Berichte aus der Mathematik

Dominik Dorsch

Stratified Optimality Theory

A Tool for the Theoretical Justification of Assumptions
in Finite-Dimensional Optimization

Shaker Verlag
Aachen 2014

Bibliographic information published by the Deutsche Nationalbibliothek
The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at http://dnb.d-nb.de.

Zugl.: D 82 (Diss. RWTH Aachen University, 2014)

Printed in Germany.

ISBN 978-3-8440-2671-9
ISSN 0945-0882

Shaker Verlag GmbH • P.O. BOX 101818 • D-52018 Aachen
Phone: 0049/2407/9596-0 • Telefax: 0049/2407/9596-9
Internet: www.shaker.de • e-mail: info@shaker.de

The minimization or maximization of a function subject to constraints is a fundamental problem which occurs in many sciences like biology, chemistry, and physics, as well as in applied fields like economics, finance, and engineering. Thus, the systematic study of Nonlinear Programs has naturally had an immense impact on those disciplines. However, over the last decades it became evident that specific structural properties of the problems arising in applications call for problem tailored mathematical optimization classes which represent these properties satisfactory. Nowadays, comprehensive theories provide necessary and sufficient optimality conditions for different optimization classes. In addition, the treatment of optimality in a broader setting is today subsumed under the field of „Variational Analysis“.

This dissertation is concerned with the question whether assumptions being imposed by different optimality theories can be considered to be „mild“ in some precise mathematical way. This is motivated by the fact that, in practice, it is impossible to verify assumptions at the (yet unknown) solutions and, hence, it would be desirable to guarantee that they are at least fulfilled for a „sufficiently“ rich set of problem instances. In order to answer the question we endow the given constraint set with a stratification, i.e., a partition into manifolds. This additional geometric structure opens the field for results from Differential Topology and, as a consequence, we are able to prove optimality conditions which hold for a dense and open subset of problem instances. The presented theory is developed in terms of classical objects from Variational Analysis like tangent and normal cones. However, the given stratification enables us, furthermore, to introduce new objects which are specifically tailored to stratified sets and, thus, have stronger properties than the classical ones.

We apply our theory exemplary to Nonlinear Semidefinite Programming, Mathematical Programs with Vanishing Constraints, and Generalized Nash Equilibrium Problems. Our geometric point of view helps us to gain new insights about structural properties of these particular problem classes. Finally we are even able to define new local solution algorithms with promising convergence properties.