

Space-Time Tree-Based Adaptive Wavelet Methods for Parabolic PDEs

Florian Stapel

Abstract

This diploma thesis is concerned with the numerical treatment of linear and parabolic partial differential equations. It is based upon an Ansatz by C. Schwab and R. Stevenson that presents a space-time weak formulation of the parabolic problem and reformulates it as an infinite sequence-space problem by the aid of a suited wavelet basis. Based upon the sequence space problem an adaptive solution method in asymptotically optimal complexity, that avoids time-stepping, was given.

We exploit this Ansatz in a framework that has been well-established for elliptic PDEs. This framework includes the usage of isotropic tensor product wavelet bases which have some theoretical and practical benefits for the numerical treatment of operator equations, even though the function spaces in use have an anisotropic structure. Within this thesis we are especially given the possibility to adapt a software package called AWM Toolbox that has been developed by J. Vorloepper in his dissertation in 2009. We provide a homogenized version of the problem formulation that is beneficial for the solution within the theoretical framework of the AWM Toolbox. Numerical results provided with our implementation confirm convergence rates that are expectable for adaptive wavelet methods of elliptic problems.

From a theoretical perspective, the thesis provides a consistent elaboration of the functional analytical aspects of space-time weak formulations. We elaborate details about the tensor product of separable Hilbert spaces and prove a novel result about the tensor product of isometries. The aforementioned homogenization of the parabolic problem is a further highlight.

Within the thesis, algorithms and concepts for the adaptive solution of the sequence space problem such as tree-structured index sets and best N-term approximations are introduced. We provide condition numbers for the operator in isotropic wavelet coordinates and present a modified solution scheme for the nonsymmetric system which is based upon a restarted GMRES method. Numerical results for different smooth and non-smooth examples conclude the elaboration.

Keywords: Adaptive Wavelet Methods, Tree Structure, Parabolic PDEs, Space-Time Weak Formulations, Homogenization, Coarsening
