

Lehrstuhl für Netzwerktheorie und Signalverarbeitung  
Technische Universität München

# **Information-theoretic Analysis and Signal Processing Techniques for Quantized MIMO Communications**

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