Nonlinear Finite Element Modeling and Analysis of Coupled Multi-Physics Smart Composite Structures

Nichtlineare Finite Elemente Modellierung und Berechnung gekoppelter Multiphysik-Probleme intelligenter Strukturen aus Verbundwerkstoffen

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Abstract

From the exhaustive literature on the state of the art of nonlinear static and dynamic finite element analysis dealing with coupled multi-physics problems of plate and shell structures, it can be observed that the majority of works are available either in the form of simplified nonlinear finite elements for piezoelectric structures or in the form of weak coupling of multi-fields with linear constitutive relations for thermopiezomechanical problems. Also not many works are available for nonlinearly coupled thermopiezomechanical finite element analysis of shell structures. Concerning magnetoelectroelastic problems, it is worth to note that so far there is no available literature on fully geometrically nonlinear analysis and 2-D shell elements for smart magnetoelectroelastic layered plates and shells. However, in order to design appropriate control systems for actuator and sensor applications, the influence of finite deformations and rotations as well as true coupling of various fields cannot be neglected.

Therefore, the objective of this thesis is to develop fully nonlinear finite element models for smart thermopiezomechanical and magnetoelectroelastic composite layered shell structures. Nonlinearly coupled thermopiezomechanical constitutive relations are used for piezoelectric structures, whereas three-way coupled constitutive relations are used to deal with the problems of magnetoelectroelasticity. The geometrically nonlinear strain-displacement relations with finite rotations in the framework of the Reissner-Mindlin first-order transverse shear deformation hypothesis are adopted for both types of finite elements. Further, numerical models are developed also for smart functionally graded structures. In the case of magnetoelectroelastic finite elements, a detailed derivation of a thermodynamically consistent theory including fully coupled magnetoelectroelastic field equations and constitutive relations is given. The developed numerical models are deployed in various examples of nonlinear statics and dynamics of beams, plates and shells, including also control problems of smart structures, to validate and to show the potentiality as well as the predictive capabilities. Additionally, the presented results are compared with those available in literature as far as possible.

Zusammenfassung

Ein Studium der umfangreichen Literatur über den Stand der Forschung auf dem Gebiet der nichtlinearen statischen und dynamischen FE Simulation von Platten und Schalen bei gekoppelten Mehrfeld-Problemen zeigt, dass die Mehrheit der Forschungen entweder in der Form vereinfachter nichtlinearer Finite Elemente oder in der Form schwacher Kopplung der Felder mit linearen konstitutiven Gesetzen für thermo-piezo-mechanische Probleme vorliegt. Nur wenige Arbeiten liegen für nichtlinear gekoppelte thermo-piezo-mechanische Berechnungen von Schalen vor. Hinsichtlich magneto-elektro-elastischer Probleme ist anzumerken, dass derzeit keine Arbeiten zur vollständig geometrisch nichtlinearen Berechnung und zu 2-D Schalenelementen für intelligente Platten und Schalen mit magneto-elektro-elastischen Schichten vorliegen. Beim Entwurf geeigneter Regelungssysteme für Aktor- und Sensoranwendungen kann der Einfluss großer Verformungen und die Kopplung der unterschiedlichen Felder jedoch nicht vernachlässigt werden.

Deshalb ist das Ziel der vorliegenden Arbeit, vollständig nichtlineare Finite Elemente Modelle für intelligente Schalenstrukturen mit thermo-piezo-mechanischen und magneto-elektro-elastischen Schichten zu entwickeln. Nichtlinear gekoppelte thermo-piezo-mechanische konstitutive Beziehungen werden für piezoelektrische Strukturen verwendet, während konstitutive Beziehungen mit Drei-Wege-Kopplung für magneto-elektro-elastische Probleme verwendet werden. Für beide Finite Element-Typen werden geometrisch nichtlineare Dehnungs-Verschiebungs-Beziehungen für finite Rotationen im Rahmen der Reissner-Mindlin Schubdeformationstheorie 1. Ordnung benutzt. Außerdem werden auch Finite Elemente Modelle für intelligente Strukturen aus funktional gradierten Werkstoffen entwickelt. Für magneto-elektro-elastische Finite Elemente Modelle wird die detaillierte Herleitung einer thermodynamisch konsistenten Theorie mit vollständig gekoppelten Feldgleichungen und konstitutiven Beziehungen durchgeführt. Die entwickelten numerischen Modelle werden in einer Vielzahl von Beispielen zur nichtlinearen statischen und dynamischen Berechung von Balken, Platten und Schalen, einschließlich Regelungsproblemen intelligenter Strukturen, eingesetzt, um sie zu validieren und ihre Berechnungsmöglichkeiten zu demonstrieren. Außerdem werden die Ergebnisse, wann immer möglich, mit in der Literatur vorliegenden Resultaten verglichen.

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