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Sensitivity Methods for Analysis and Design of Dynamic Systems with Applications in Control Engineering

Feedforward Control – Feedback Control –
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Preface

This Habilitation Thesis originates from my scientific work as a Research Associate with the Chair of Mechatronics at the Faculty of Mechanical Engineering and Marine Technology at the University of Rostock, Germany.

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Rostock, August 2017 Andreas Rauh

As far as the laws of mathematics refer to reality, they are not certain;
and as far as they are certain, they do not refer to reality

ALBERT EINSTEIN (1879-1955)

Sensitivity Methods for Analysis and Design of Dynamic Systems with Applications in Control Engineering

Feedforward Control • Feedback Control • Robust Control • State Estimation

In this work, a unified approach is presented for the sensitivity analysis of dynamic systems and for the use of the computed sensitivity values in selected applications. These applications follow the aim of designing novel robust and efficient control as well as state and parameter estimation approaches.

Differential sensitivities represent partial derivatives of the state trajectories of a dynamic system with respect to at least one of the following quantities:

- initial conditions of the state vectors,
- system parameters,
- control inputs, and
- disturbance variables.

It has to be pointed out that differential sensitivities can be computed for dynamic system models that are either stated as ordinary differential equations, differential-algebraic equations, partial differential equations, or finite-dimensional sets of difference equations. With the help of the computed differential sensitivities, small-signal information can be determined concerning the influence of the above-mentioned quantities on the state trajectories. This small-signal nature is characteristic for the sensitivity computation in classical floating point arithmetic.

However, extensions of the notion of sensitivity to finitely large parameter intervals become possible if the sensitivity analysis is performed by using techniques from interval analysis. Then, guaranteed enclosures are obtained for each of the entries of the required sensitivity vectors. Hence, the corresponding interval bounds contain the true point-valued sensitivity values for each possible initial condition, parameter, control input, or disturbance value from a predefined interval.

This type of interval extension provides valuable information that can be exploited for the verified analysis of the dynamics of a system model by using simulation tools from interval analysis. Moreover, interval evaluations of sensitivities can be used if robust extensions of control strategies are of interest, for example, the predictive control approaches mentioned in the following.

Besides a generalization of sensitivity analysis to systems with bounded parameter uncertainty, the following methodological aspects are studied in detail within this work:

- Analysis of the sensitivity of the open-loop dynamics of a control system
- Analysis of the sensitivity of the closed-loop dynamics of a control system
- Implementation of desensitizing state and output feedback controllers for continuous-time and discrete-time processes
- Implementation of state and sensitivity estimators for continuous-time and discrete-time processes
- Implementation of predictive control strategies that are solved online by including sensitivity information in gradient descent methods or Newton-like approaches
- Implementation of state and parameter estimation strategies as the dual task to predictive control
- Extension of predictive control procedures to learning-type control approaches as well as to techniques for the offline feedforward control synthesis
- Use of eigenvalue and eigenvector sensitivities in extended linearization approaches in a framework for feedback control and state observation

To demonstrate and validate the usability of the above-mentioned methodological sensitivity-based approaches in practical control tasks, a broad variety of open-loop and closed-loop control systems as well as state and parameter estimation problems are investigated.

These tasks are listed in the following with a focus on the specific features that can be achieved by means of a sensitivity analysis:

1. *Computation of verified state enclosures for a catalytic reaction process by using interval-based initial value problem solvers.* The conservatism of interval arithmetic techniques often leads to a blow-up of the width of the state enclosures in simulations of dynamic systems. However, this effect can commonly be eliminated fully for cooperative systems, if they show a monotone dependency on interval parameters. Cooperativity can be proven rigorously by a verified simulation of the associated sensitivity equations.
2. *Desensitizing feedback control design.* Desensitizing feedback control strategies are a possibility to enhance the robustness of closed-loop control systems against inevitable identification inaccuracies of selected system parameters. This property is exploited for the control design of a finite-volume model of a spatially two-dimensional heat transfer process. The same also holds for the control of a simplified robotic manipulator in a further benchmark application.
3. *Sensitivity-based estimation of states and parameters in a moving horizon framework.* States and parameters can be estimated efficiently by using sensitivity-based procedures. This is demonstrated for the dynamic system model of a Lithium-Ion battery and for the thermal behavior of a high-temperature solid oxide fuel cell. In contrast to classical estimation techniques, based on an (Extended) Kalman Filter, the presented approaches are easier to parameterize and naturally guarantee that slowly varying parameters are adapted slower than rapidly changing state variables. Moreover, the corresponding program code is applicable to estimating states both online during system operation and offline during pure identification phases.
4. *Sensitivity-based closed-loop predictive control.* Predictive control procedures make use of a moving window of forecasted future system states and outputs. Corresponding algorithms can be implemented with the help of sensitivity-based approaches by using both classical floating point arithmetic and interval analysis. Using interval techniques, it becomes possible to implement controllers that prevent the violation of control, state, and output constraints despite bounded uncertainty of selected system parameters and disturbance variables. Implementations are shown for the control of a spatially one-dimensional heat transfer process (described by a finite volume and finite element semi-discretization), for the temperature control of a climate chamber, and for the thermal behavior of a fuel cell stack module.
5. *Extensions of predictive control procedures to a learning-type control approach.* Learning-type controllers are useful for accurate control if the repeated tracking of identical predefined trajectories is of interest. Then, the control errors can usually become smaller than the optimal control performance of a pure state feedback controller. The use of sensitivity-based approaches is demonstrated in this context for two types of distributed heating systems as well as for trajectory tracking and oscillation attenuation of a flexible rack feeder structure. In a pure offline evaluation, these techniques provide a means to determine feedforward control sequences for cases in which analytic solutions are not possible, for example, in cases in which a dynamic system is not differentially flat. For the online application, iterative control approaches are obtained, which stabilize the system dynamics during a single control execution and which reduce the tracking errors from one execution to the next. In analogy to the predictive control design, it is possible to describe the system inputs either by piecewise constant signals or by polynomial expressions. The latter ones are advantageous for long control horizons and for cases in which the numbers of design parameters in piecewise constant parameterizations may become prohibitively large.
6. *Extended linearization by incremental gain scheduling.* Extended linearization techniques can be used to design control approaches with variable gain values which structurally resemble linear state feedback techniques. Here, the controller gains have to be adapted along the tra-

jectories of a closed-loop control system if the system and input matrices depend noticeably on the state trajectories. To make these adaptation techniques efficient for systems with a non-negligible state dependency in the system and input matrices, gain adaptation schemes are determined in this work which exploit the information of eigenvalue and eigenvector sensitivities against variations in the before-mentioned matrices. The presented applications comprise the stabilization of an inverted pendulum as well as trajectory tracking control for ship motions.

Sensitivitätsbasierte Methoden für die Analyse sowie den Entwurf von dynamischen Systemen mit Anwendungen in der Regelungstechnik

Vorsteuerung • Regelung und Zustandsrückführung • Robuste Regelung • Zustandsschätzung

In der vorliegenden Arbeit wird ein einheitlicher Ansatz zur Sensitivitätsanalyse dynamischer Systeme und zur Anwendung der berechneten Sensitivitätswerte vorgestellt. Bei diesen Anwendungen wird das Ziel verfolgt, neuartige robuste und effiziente Regelungsstrategien sowie Zustands- und Parameterschätzverfahren zu implementieren.

Differentielle Sensitivitäten stellen partielle Ableitungen der Zustandstrajektorien eines dynamischen Systems gegenüber Variationen in mindestens einer der folgenden Größen dar:

- Anfangsbedingungen des Zustandsvektors,
- Systemparameter,
- Stellgrößen und
- Störgrößen.

Es ist dabei hervorzuheben, dass differentielle Sensitivitäten für dynamische Systemmodelle berechnet werden können, welche entweder als gewöhnliche Differentialgleichungen, differential-algebraische Gleichungen, partielle Differentialgleichungen oder endlich-dimensionale Systeme von Differenzengleichungen beschrieben sind. Mittels der berechneten differentiellen Sensitivitäten gelingt es, eine Kleinsignalinformation hinsichtlich des Einflusses der oben genannten Größen auf die Zustandstrajektorien darzustellen. Dieser Kleinsignalcharakter tritt insbesondere zu Tage, wenn die Berechnung von Sensitivitäten mittels klassischer Gleitkomma-Arithmetik erfolgt.

Eine Verallgemeinerung des Sensitivitätsbegriffs wird durch die Verwendung von Techniken der Intervallarithmetik zur Sensitivitätsberechnung möglich. Damit lassen sich Erweiterungen auf endlich große Parameterintervalle erreichen. Hieraus ergeben sich garantierte Einschlüsse für die Gesamtheit aller Einträge der benötigten Sensitivitätsvektoren. Daher beinhalten die entsprechenden Intervallschranken die wahren punktförmigen Sensitivitätswerte für jeden möglichen Anfangswert, Parameter, Stelleingriff oder Störgrößenwert innerhalb des jeweils vordefinierten Intervalls.

Diese Art der Intervallerweiterung stellt eine nützliche Information hinsichtlich einer verifizierten Analyse der Systemdynamik unter Verwendung intervallarithmetischer Simulationsverfahren zur Verfügung. Zusätzlich können diese Sensitivitätsintervalle im Rahmen robuster Regelungen genutzt werden, so zum Beispiel für die nachfolgend erwähnten prädiktiven Regelungsansätze.

Neben einer Verallgemeinerung der Sensitivitätsanalyse auf Systeme mit wertebereichsbeschränkten Parametern werden in dieser Arbeit die folgenden methodischen Aspekte im Detail betrachtet:

- Sensitivitätsanalyse der Dynamik von offenen Regelkreisen
- Sensitivitätsanalyse der Dynamik von geschlossenen Regelkreisen
- Implementierung von Zustands- und Ausgangsrückführungen für zeitkontinuierliche und zeitdiskrete Prozesse mit dem Ziel einer Empfindlichkeitsreduktion gegenüber Unsicherheiten
- Implementierung von Zustands- und Sensitivitätsschätzern für zeitkontinuierliche und zeitdiskrete Prozesse
- Implementierung prädiktiver Regelungsstrategien, welche online mittels Sensitivitätsinformation in Gradienten-Abstiegs- oder Newton-ähnlichen Verfahren gelöst werden
- Implementierung von Zustands- und Parameterschätzverfahren als duales Problem zur prädiktiven Regelung
- Erweiterung von prädiktiven Regelungsansätzen auf Regelungen mit lernendem Verhalten sowie auf Verfahren zur offline Steuerungsberechnung
- Verwendung von Eigenwert- und Eigenvektorsensitivitäten in Ansätzen der erweiterten Linea-

risierung für die Zustandsregelung und Zustandsschätzung

Um die Anwendbarkeit der obigen sensitivitätsbasierten Ansätze in praktischen Regelungsaufgaben zu veranschaulichen und zu validieren, werden für eine Vielzahl von Systemen Fragestellungen der Steuerung und Regelung sowie der Zustands- und Parameterschätzung untersucht.

Diese Fragestellungen sind nachfolgend aufgelistet, wobei die hauptsächlich durch eine Sensitivitätsanalyse erreichten Eigenschaften in den Fokus gestellt werden:

1. *Berechnung verifizierter Zustandseinschlüsse für einen katalytischen Reaktionsprozess unter Verwendung intervallarithmetischer Anfangswertproblem-Löser.* Oft resultiert ein Aufblähen von Zustandseinschlüssen aus der Konservativität von Intervallmethoden in der Simulation dynamischer Systeme. Dieser Effekt kann jedoch häufig für kooperative Systeme vollständig verhindert werden, wenn diese Systeme eine monotone Abhängigkeit in Bezug auf Intervallparameter aufweisen. Ein strikter Nachweis der Kooperativität lässt sich dabei durch eine verifizierte Simulation der entsprechenden Sensitivitätsgleichungen erreichen.
2. *Sensitivitätsreduktion durch Zustandsrückführung.* Eine Sensitivitätsreduktion durch zustandsrückführende Regelungen führt zu einer Steigerung der Robustheit gegenüber unvermeidlichen Ungenauigkeiten in der Identifikation ausgewählter Systemparameter. Diese Eigenschaft wird für ein Finite-Volumen-Modell eines örtlich zwei-dimensionalen Wärmeleitungsprozesses veranschaulicht. Ebenso gilt dies für ein weiteres Beispiel, welches sich mit der Regelung eines vereinfachten Robotermanipulators befasst.
3. *Sensitivitätsbasierte Schätzung von Zuständen und Parametern im Rahmen von Verfahren mit einem gleitenden Zeitfenster.* Zustände und Parameter lassen sich effizient durch sensitivitätsbasierte Ansätze schätzen. Dies wird für das dynamische Modell einer Lithium-Ionen-Batterie sowie für das thermische Verhalten einer Feststoff-Oxid-Brennstoffzelle veranschaulicht. Im Vergleich zu klassischen Schätzverfahren, welche auf dem (Erweiterten) Kalman-Filter basieren, sind die vorgestellten Ansätze leichter zu parametrieren und stellen auf natürliche Weise sicher, dass langsam veränderliche Parameter langsamer als schnell veränderliche Zustandsgrößen in der Schätzung adaptiert werden. Zusätzlich lässt sich der entsprechende Programmcode sowohl online während des Betriebs eines Systems zur Zustandsschätzung als auch offline zur reinen Identifikation nutzen.
4. *Sensitivitätsbasierte prädiktive Regelungen.* Prädiktive Regelungen nutzen für die Stellgrößenberechnung ein gleitendes Zeitfenster vorhergesagter, in der Zukunft liegender Zustände und Systemausgänge. Entsprechende Algorithmen lassen sich bei der Verwendung sensitivitätsbasierter Verfahren sowohl mittels klassischer Gleitkomma-Arithmetik als auch unter Zuhilfenahme von Intervallmethoden umsetzen. Intervallmethoden ermöglichen es dabei, Regelungen zu realisieren, welche eine Verletzung von Stellgrößen-, Zustands- und Ausgangsbeschränkungen trotz wertebereichsbeschränkter, unsicherer Parameter und Störgrößen in gesicherter Weise verhindern. Ergebnisse werden für die Regelung eines örtlich ein-dimensionalen Wärmeleitungssystems (beschrieben über Finite-Volumen- und Finite-Elemente-Semidiskretisierungen), für die Temperaturregelung einer Klimakammer sowie für die Regelung des thermischen Verhaltens eines Brennstoffzellen-Stackmoduls dargestellt.
5. *Erweiterung prädiktiver Regelungen hin zu lernenden Verfahren.* Sofern sich Führungsgrößen durch zyklisch wiederkehrende Trajektorien beschreiben lassen, können Regelungen mit lernender Charakteristik für eine präzise Trajektorienfolgeregelung implementiert werden. Dabei lassen sich die Regelfehler im Vergleich zu einer optimalen Einstellung reiner Zustandsrückführungen von einer Durchführung zur nächsten weiter reduzieren. Die Verwendung von sensitivitätsbasierten Ansätzen wird in diesem Zusammenhang für zwei örtlich verteilte Wärmeleitungsprozesse sowie für ein elastisches Regalbediensystem aufgezeigt. In einer reinen Offline-

Umsetzung erlauben diese Verfahren die numerische Berechnung von Vorsteuerungen. Dies gilt auch in Fällen, in denen eine analytische Lösung aufgrund fehlender differentieller Flachheit nicht möglich ist. In der Online-Umsetzung führen diese Regelungsverfahren zu einer Stabilisierung der Systemdynamik innerhalb einer einzelnen Ausführung der Regelungsaufgabe sowie zu einer sukzessiven Verbesserung der Regelgüte von einer Ausführung zur nächsten. In Analogie zu prädiktiven Regelungen können Stellgrößen hier entweder als stückweise konstant angenommen oder über polynomiale Ausdrücke parametriert werden. Die Nutzung polynomialer Ausdrücke ist vorteilhaft, wenn die Anzahl von Entwurfsfreiheitsgraden bei einer abschnittsweise konstanten Beschreibung über lange Zeitspannen unverhältnismäßig groß wird.

6. *Erweiterte Linearisierung für eine gesteuerte Parameteradaption.* Strategien zur Adaption der Parameter von Zustandsreglern, die strukturell zu linearen Ansätzen ähnlich sind, lassen sich durch Verfahren der erweiterten Linearisierung herleiten. Hierbei müssen die Reglerverstärkungen aufgrund von Zustandsabhängigkeiten der System- und Stelleingriffsmatrizen an die jeweiligen Systemtrajektorien des geschlossenen Regelkreises angepasst werden. In dieser Arbeit erfolgt für Systeme mit nicht vernachlässigbaren Zustandsabhängigkeiten der System- und Stelleingriffsmatrizen eine sensitivitätsbasierte Parameteradaption. Diese nutzt Information über Variationen der Eigenwerte und Eigenvektoren in Bezug auf die Einträge der voran genannten Matrizen. Aufgezeigte Anwendungen umfassen die Stabilisierung eines inversen Pendels sowie die Trajektorienfolgeregelung von Schiffen.

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List of Symbols

The following list of symbols provides an overview of the notation used in this work. As a general convention, small letters are reserved for (real) scalars (e.g. $a \in \mathbb{R}$) and vectors (e.g. $\mathbf{x}$) and capital letters (e.g. $\mathbf{A}$) for matrices. Vectors and matrices are generally typeset in boldface letters.

Moreover, intervals are denoted explicitly by square brackets, e.g., $[\mathbf{x}] = \begin{bmatrix}[x_1] & \dots & [x_n]\end{bmatrix}^T$ represents an interval vector with $[x_i] = [\underline{x}_i \,;\, \overline{x}_i]$, $\underline{x}_i \leq x_i \leq \overline{x}_i$, $i = 1, \dots, n$.

Note that the following list is structured according to the two major parts of this work, namely, methodological approaches and simulations as well as experiments of selected application scenarios. Further not explicitly listed supplementary, intermediate, and auxiliary variables that are used in this work are specified at the place of their first usage.

Methodological Procedures

Fundamental Notation, System Parameters, State, Input and Output Variables

$\mathbf{P} \succ 0$	Positive definiteness of a matrix $\mathbf{P}$
$\mathbf{P} \succeq 0$	Positive semi-definiteness of a matrix $\mathbf{P}$
$\mathbf{P} \prec 0$	Negative definiteness of a matrix $\mathbf{P}$
$\mathbf{P} \preceq 0$	Negative semi-definiteness of a matrix $\mathbf{P}$
$\mathcal{D}$	Polytopic (vertex-related) representation of an uncertain dynamic system
$\mathcal{D}(\cdot)$	Performance criterion for sensitivity-based control and estimation
$\mathcal{D}_{\mathrm{L}}(\cdot)$	Linear performance criterion for sensitivity-based control and estimation
$\mathcal{D}_{\mathrm{Q}}(\cdot)$	Quadratic performance criterion for sensitivity-based control and estimation
$\mathbb{R}$	Set of real numbers
$\mathbf{A}$	System matrix of a continuous-time dynamic system
$\mathbf{A}(\mathbf{x}(t))$	System matrix in quasi-linear state-space representation
$\mathbf{A}_{\mathrm{C}}(\mathbf{x}(t), \mathbf{K})$	System matrix in closed-loop control of a quasi-linear state-space representation
$\mathbf{A}_{\mathrm{O}}(\hat{\mathbf{x}}(t), \mathbf{H})$	System matrix of an observer in quasi-linear state-space representation
$\mathbf{B}$	Input matrix of a continuous-time dynamic system
$\mathbf{B}(\mathbf{x}(t))$	Input matrix in quasi-linear state-space representation
$\mathbf{C}$	Output matrix of a continuous-time or discrete-time dynamic system
$\mathbf{C}(\mathbf{x}(t))$	Output matrix in quasi-linear state-space representation
$\mathbf{D}$	Feed-through matrix of a continuous-time or discrete-time dynamic system
$\mathbf{D}(\mathbf{x}(t))$	Feed-through matrix in quasi-linear state-space representation
$\mathbf{e}(t)$, $\mathbf{e}(t_k)$	Error vectors in control and observer design
$\mathbf{f}(\cdot)$	General state-space representation of a continuous-time dynamic system
$\dot{\mathbf{x}} = \mathbf{f}(\cdot)$, $\mathbf{0} = \mathbf{g}(\cdot)$	Coupling of first-order differential equation(s) and algebraic (output) constraint(s)

$F(\cdot)$	General representation of a partial differential equation with the distributed storage variable $\vartheta_{\mathcal{I}}(\mathbf{x},t)$ (differentiation multi-index $\mathcal{I}$)
$\mathbf{F}(\cdot)$	General representation of a set of differential-algebraic equations with the differential and algebraic states $\mathbf{x}(t)$ and $\mathbf{y}(t)$, resp.
$\mathfrak{g}(\mathfrak{x}(t), \mathbf{u}(t))$	Augmented output constraint for sets of differential-algebraic equations
$\mathbf{H}$	Feedback gain matrix of a continuous-time state observer
$\mathbf{I}$	Identity matrix
J_c, J_d	Performance criterion for optimal control design of continuous-time and discrete-time systems
$\mathbf{K}$	Controller gain matrix of a continuous-time dynamic system (full state feedback)
$\mathbf{K}(\mathbf{x}(t))$	State-dependent controller gain matrix
$\mathbf{K}_\mathrm{P}$, $\mathbf{K}_\mathrm{I}$	Proportional and integral controller gain matrices of a continuous-time dynamic system
$\mathbf{K}_\mathrm{o}$	Controller gain matrix of a continuous-time dynamic system (output feedback)
$L_\mathbf{f}\mathbf{h}(\mathbf{x})$	Lie derivative of a differentiable (nonlinear) expression $\mathbf{h}(\mathbf{x})$ in the direction of the vector field $\mathbf{f}$
N	Prediction horizon in sensitivity-based control as well as window length for sensitivity-based estimation
N_p	Pass length for sensitivity-based learning control
$p(s)$	Characteristic polynomial
$\mathbf{p}$	Vector of (uncertain) system parameters
$\mathbf{q}(\mathbf{x})$, $\mathbf{Q}(\mathbf{x})$	State-dependent observability mapping and observability matrix
$[\mathbf{R}]$	Guaranteed error bound in VALENCIA-IVP
s	Laplace variable
$\mathbf{S}_\mathrm{ff}$	Feedforward gain matrix of a continuous-time feedback controller
$\mathbf{S}(\mathbf{x}(t))$	State-dependent feedforward gain matrix
$\mathbf{s}_i(t) \in \mathbb{R}^{n_\mathrm{x}}$, $\mathbf{q}_i(t) \in \mathbb{R}^{n_\mathrm{x}}$	i-th differential sensitivity vector of a continuous-time dynamic system (exact/ approximated)
$\mathbf{s}_i(t_k) \in \mathbb{R}^{n_\mathrm{x}}$, $\mathbf{q}_i(t_k) \in \mathbb{R}^{n_\mathrm{x}}$	i-th differential sensitivity vector of a discrete-time dynamic system (exact/ approximated)
ΔT	Discretization step size
$\mathbf{u}(t) \in \mathbb{R}^{n_\mathrm{u}}$	Control vector of a continuous-time dynamic system
$\mathbf{u}(t_k) \in \mathbb{R}^{n_\mathrm{u}}$	Control vector of a discrete-time dynamic system
$\Delta\mathbf{u}(t) \in \mathbb{R}^{n_\mathrm{u}}$	Control increment for predictive control of a continuous-time dynamic system
$\Delta\mathbf{u}(t_k) \in \mathbb{R}^{n_\mathrm{u}}$	Control increment for predictive control of a discrete-time dynamic system
$\mathbf{u}_\mathrm{fb}(t)$	Feedback control vector of a continuous-time dynamic system
$\mathbf{u}_\mathrm{fb}(t_k)$	Feedback control vector of a discrete-time dynamic system
$\mathbf{u}_\mathrm{ff}(t)$	Feedforward control vector of a continuous-time dynamic system
$\mathbf{u}_\mathrm{ff}(t_k)$	Feedforward control vector of a discrete-time dynamic system
$V(\mathbf{x}(t))$, $V(\mathbf{x}(t_k))$	Lyapunov function (candidate)
$\mathbf{v}_i$, $\mathbf{w}_i$	i-th (left/ right) eigenvector of a dynamic system (generally complex-valued)
$\mathbf{w}(t) \in \mathbb{R}^{n_\mathrm{w}}$	Reference signal vector of a continuous-time dynamic system
$\mathbf{w}(t_k) \in \mathbb{R}^{n_\mathrm{w}}$	Reference signal vector of a discrete-time dynamic system
$\mathbf{x}(t) \in \mathbb{R}^{n_\mathrm{x}}$	State vector of a continuous-time dynamic system

$\mathbf{x}^*(t) \in \mathbb{R}^{n_\mathrm{x}}$	Exact solution to an initial value problem for ordinary differential equations
$\tilde{\mathbf{x}}(t) \in \mathbb{R}^{n_\mathrm{x}}$	Non-verified approximate solution to an initial value problem for ordinary differential equations
$[\mathbf{x}_\mathrm{e}](t)$	Verified exponential state enclosure
$\mathbf{x}(t_k) \in \mathbb{R}^{n_\mathrm{x}}$	State vector of a discrete-time dynamic system
$\hat{\mathbf{x}}(t)$, $\hat{\mathbf{y}}(t)$	State/ output estimate determined by a continuous-time observer
$\hat{\mathbf{x}}(t_k)$, $\hat{\mathbf{y}}(t_k)$	State/ output estimate determined by a discrete-time observer
$\mathfrak{x}(t)$	Augmented state vector for sets of differential-algebraic equations
$\mathbf{y}(t) \in \mathbb{R}^{n_\mathrm{y}}$	Output vector of a continuous-time dynamic system
$\mathbf{y}(t_k) \in \mathbb{R}^{n_\mathrm{y}}$	Output vector of a discrete-time dynamic system
$\mathbf{y}_\mathrm{d}(t) \in \mathbb{R}^{n_\mathrm{y}}$	Vector of desired system outputs of a continuous-time dynamic system
$\mathbf{y}_\mathrm{d}(t_k) \in \mathbb{R}^{n_\mathrm{y}}$	Vector of desired system outputs of a discrete-time dynamic system
$\mathbf{y}_\mathrm{m}(t) \in \mathbb{R}^{n_\mathrm{m}}$	Measured output vector of a continuous-time dynamic system (analogously the index m is used for all further occurrences in which measured and general system outputs have to be distinguished)
$\mathbf{y}_\mathrm{m}(t_k) \in \mathbb{R}^{n_\mathrm{m}}$	Measured output vector of a discrete-time dynamic system
α_c	Step-size control factor in sensitivity-based (predictive) control approaches
α_e	Step-size control factor in sensitivity-based estimation approaches
$\alpha(t)$	Time scaling parameter for path following approaches
γ, γ_O	Stability margin in control and observer design
κ	Iteration counter
λ_i	i-th eigenvalue of a dynamic system (generally complex-valued)
η_y	Output error scaling factor in the design of proportional and integrating state feedback controllers
$\mathbf{\Phi}$	System matrix of a discrete-time dynamic system
$\mathbf{\Phi}(\cdot)$	General state-space representation of a discrete-time dynamic system
$\mathbf{\Psi}$	Input matrix of a discrete-time dynamic system
$\mathbf{\Pi}$	Controller gain matrix of a discrete-time dynamic system (full state feedback)
$\mathbf{\Pi}_\mathrm{P}$, $\mathbf{\Pi}_\mathrm{I}$	Proportional and integral controller gain matrices of a discrete-time dynamic system
$\mathbf{\Pi}_\mathrm{o}$	Controller gain matrix of a discrete-time dynamic system (output feedback)
$\vartheta_\mathcal{I}(\mathbf{x},t)$, $\varsigma_\mathcal{I}(\mathbf{x},t)$	Storage and sensitivity variables of a partial differential equation
$\mathbf{\Theta}$	Feedback gain matrix of a discrete-time state observer
ζ	Number of subsequent constant control steps in sensitivity-based predictive and learning-type procedures
$\boldsymbol{\xi}$	Generalized parameter vector of a dynamic system

Application Scenarios

Catalytic Reactor

k_1, k_2, k_3	System parameters (reaction rates)
$u(t)$	Catalyst concentration
$x_1(t)$, $x_2(t)$, $x_3(t)$	Concentrations of the substances A_1, A_2, A_3
$\mathbf{z}(t)$	Transformed state vector

Simple Robotic Manipulator

g	Gravitational acceleration
k_f	Friction coefficient
L	Length of the robot arm
m	Mass parameter
$x_1(t)$	Angle (first state variable)
$x_2(t)$	Angular velocity (second state variable)

One-Dimensional Heating System

A_AE, A_RC, A_RE	Cross section areas
b_AE, b_RE, h_AE, h_RE l_AE, l_RE	Geometric length parameters (resp. b, d, l)
$b_{i,k,M}(z)$	Space-dependent Bernstein polynomial (order M, defined for the i-th element)
V_AE, V_RE	Volume of respective elements
c_A, c_p, c_R	Specific heat capacities
K_1, $p_1, \ldots, p_5$	Lumped system parameters
$\dot{m}(t)$	Air mass flow
$\dot{Q}_{\mathrm{H}i}$	Heat flow of the i-th Peltier element
$q(z,t)$	Spatially distributed heat flux density
$q_{i,k,M-1}(t)$	Time-dependent coefficients of Bernstein polynomial heat flux density approximation (order $M-1$, defined for the i-th element)
$\mu(z,t)$	External control and disturbance input
$\mu_i(z,t)$	External control and disturbance input in the segment i ($i = 1,\ldots,4$)
ρ	Volume density of the rod material
ρ_A, ρ_R	Volume densities (air, rod)
$\vartheta(z,t)$	Spatially distributed temperature
$\bar{\vartheta}(z,t)$	Overtemperature $\bar{\vartheta}(z,t) = \vartheta(z,t) - \vartheta_\mathrm{A}$ for $\vartheta_\mathrm{A} = \mathrm{const}$
$\vartheta_\mathrm{A}(t)$	Ambient temperature
$\vartheta_\mathrm{I}(t)$	Air canal inlet temperature
$\vartheta_{i,\mathrm{FE}}(z,t)$	Rod temperatures in the finite element model ($i = 1,\ldots,4$)
$\vartheta_{i,\mathrm{FV}}(t)$	Rod temperatures in the finite volume model ($i = 1,\ldots,4$)
$\vartheta_{i,\mathrm{AC}}(z,t)$	Air canal temperatures in the finite element model ($i = 5,\ldots,8$)
$\vartheta_{i,\mathrm{FV}}(t)$	Air canal temperatures in the finite volume model ($i = 5,\ldots,8$)
$\theta_{i,k,M}(t)$	Time-dependent coefficients of Bernstein polynomial temperature approximation (order M, defined for the i-th element)
α, α_A, α_R	Convective heat transfer coefficients
λ_R	Heat conductivity

Two-Dimensional Heating System

$b_{i,k,M_\zeta}(\zeta)$	Space-dependent Bernstein polynomial (order M_ζ, defined for the i-th element)
$\mathbf{b}_{\mathcal{I},\mathcal{M}}(y,z)$	Bivariate Bernstein polynomial vector (concerning finite element modeling in the $\mathcal{I}$-th plate segment)

$c_{\mathcal{I}}$	Specific heat capacity
$E_{\mathrm{th},\mathcal{I}}$	Local thermal energy
h	Plate thickness
l_1, l_2	Lengths of plate edges
$m_{\mathcal{I}}$	Local mass parameter
N_{y}, N_{z}	Numbers of finite (volume) elements
p_u	Uncertain gain of the Peltier element heat flows
$q_y(y,z,t)$, $q_z(y,z,t)$	Spatially distributed heat flux densities in Cartesian coordinates
$\mathbf{q}_{y,\mathcal{I}}(t)$, $\mathbf{q}_{z,\mathcal{I}}(t)$	Time-dependent parameters for the approximation of the heat flux density in the $\mathcal{I}$-th plate segment
$\dot{Q}_{\mathrm{H},\mathcal{I}}$	Peltier element heat flows
$\dot{Q}_{\mathrm{hc},\mathcal{I}}^{\mathcal{J}}$	Heat conduction
$\dot{Q}_{\mathrm{conv},\mathcal{I}}$	Heat convection
α	Convective heat transfer coefficient
λ	Heat conductivity
κ_1, κ_2	Abbreviated system parameters
$\mu(y,z,t)$	External control and disturbance input
$\vartheta_{\mathrm{A}}(t)$	Ambient temperature
$\vartheta_{\mathcal{I}}(t)$	Piecewise homogeneous temperature approximation
$\vartheta(y,z,t)$	Spatially distributed temperature in Cartesian coordinates
$\boldsymbol{\theta}_{\mathcal{I}}(t)$	Time-dependent parameters for the approximation of the temperature in the $\mathcal{I}$-th plate segment

Climate Chamber

A_{HE}, A_{W}	Surface areas
c_{A}, c_{HE}	Specific heat capacities
m_{A}, m_{HE}	Mass parameters
$\dot{Q}_{\mathrm{HC}}$, $\dot{Q}_{\mathrm{HE}}$, $\dot{Q}_{\mathrm{W}}$	Heat flows
q_{HE}	Volume flow of the heat exchanger
T_{HC}	Time constant of the heating-cooling unit
$u(t)$	Control input (set-point of the heating cooling unit)
α_{HE}, α_{W}	Convective heat transfer coefficients
$\vartheta_{\mathrm{A}}(t)$	Air temperature
$\vartheta_{\mathrm{HE}}(t)$	Heat exchanger temperature
$\vartheta_{\mathrm{O}}(t)$	Ambient temperature

Lithium-Ion Battery

C_0	Nominal battery capacitance
C_{Bat}	Battery capacitance
C_i	Capacitance in equivalent circuit lag elements
c_{pBat}	Specific heat capacity
$\mathrm{d}E_{\mathrm{T}}\left(T_{\mathrm{Bat}}(t)\right)$	Temperature-dependent variation of the open-circuit voltage
$i(t)$	Terminal current
$i \in \{\mathrm{TL},\mathrm{TS}\}$	Lag elements with large/ small time constants
m_{Bat}	Battery mass

$\mathbf{Q}$, $\mathbf{R}$	Covariance matrices of process and measurement noise
R_i	Resistance in equivalent circuit lag elements
R_S	Series resistance
$T_{Bat}(t)$	Battery temperature
$T_{cool}(t)$	Temperature of cooling medium
$v_{Bat}(t)$	Terminal voltage
$v_i(t)$	Voltage across equivalent circuit lag elements
$v_{OC}(t)$	Open-circuit voltage
$\alpha(t)$, $\beta(t)$, $\gamma(t)$	Ageing parameters in the equivalent circuit model
$\mu(t)$	Capacity fading factor
η	Cooling efficiency
$\sigma(t)$	State of charge

Solid Oxide Fuel Cells (Thermodynamic Subsystem Model)

$C_{AG,i,j,k}$	Local gas heat capacity (anode gas, depending on temperature and local mass flow)
$C_{CG,i,j,k}$	Local gas heat capacity (cathode gas, depending on temperature and local mass flow)
$c_{i,j,k}$	Local specific heat capacity
F	Faraday constant
$I_{i,j,k}$	Local electric current
L, M, N	Numbers of finite volume elements in the semi-discretization approach
$m_{i,j,k}$	Local mass parameter
$\dot{m}_{CG,nom}$	Nominal cathode gas mass flow in the predictive control approach
$\dot{m}_{\zeta,i,j,k}(t)$	Local gas mass flow of the species ζ (e.g. mass flow of consumed hydrogen $\dot{m}_{H_2,i,j,k}(t)$ for $\zeta = H_2$)
M_{H_2}	Molar mass of hydrogen
$P_{El,i,j,k}(t)$	Heat flows due to Ohmic losses
$\dot{Q}_{R,i,j,k}(t)$	Heat flows due to exothermic reactions
$\dot{Q}_{\eta,i,j,k}(t)$	Heat flows due to heat conduction and convection
$R_{El,i,j,k}$	Local Ohmic resistance
$R_{th,\eta}^{i,j,k}$	Thermal resistance
$\mathbf{u}_1(t)$	System inputs related to the cathode gas enthalpy flow at the inlet manifold, $\mathbf{u}_1(t) = \begin{bmatrix} \vartheta_{CG,in}(t) & \dot{m}_{CG,in}(t) \end{bmatrix}^T$
$\mathbf{u}_2(t)$	System inputs related to the anode gas enthalpy flow at the inlet manifold, $\mathbf{u}_2(t) = \begin{bmatrix} \vartheta_{AG,in}(t) & \dot{m}_{N_2,in}(t) & \dot{m}_{H_2,in}(t) & \dot{m}_{H_2O,in}(t) \end{bmatrix}^T$
z	Number of reacting electrons
$\Delta_R H_{i,j,k}$	Temperature-dependent reaction enthalpy
κ_1, κ_2, κ_3	Weighting factors for the predictive control approach
$\vartheta_{i,j,k}(t)$	Piecewise homogeneous temperature approximation
$\vartheta_A(t)$	Ambient temperature
$\vartheta_{CG,nom}$	Nominal cathode gas temperature in the predictive control approach
ϑ_{max}	Temperature constraint in the predictive control approach
$\vartheta_P(t)$	Boundary condition with respect to the temperature at the inlet gas manifolds (specified by gas preheaters P)

Flexible Rack Feeder System

A	Beam cross section area
E	Young's modulus
$F_F(\dot{y}_0(t))$	Friction force
$F_M(t)$	Motor force
g	Gravitational acceleration
I_{zB}	Second moment of area
k_d	Damping coefficient
l	Length of the beam structure
m_0	Carriage mass
m_C	Cage mass
m_E	End mass at upper beam tip
T_{1y}	Time constant for the carriage velocity controller
$v(x_C(t),t)$	Bending deflection
$v_0(t)$	Desired carriage velocity (system input)
$v_1(t)$	Generalized coordinate for the first bending mode
$x_C(t)$	Absolute vertical cage position
$y_0(t)$	Absolute carriage position
$y_C(t)$	Absolute horizontal cage position (system output $y(t)$)
$\kappa(t)$	Normalized cage position in vertical direction
ρ	Volume density of beam material
θ_C	Mass moment of inertia of the cage

Inverted Pendulum on a Cart

a	Length of the pendulum
$F_C(t)$	Accelerating force of the carriage controller
g	Gravitational acceleration
m	Point mass at the pendulum tip
M	Carriage mass
T_1	Time constant of the underlying carriage velocity controller
$u(t)$	Desired carriage velocity (system input)
$y(t)$	Horizontal position of the pendulum tip (system output)
$z(t)$	Horizontal carriage position
$\alpha(t)$	Pendulum angle

Ship Tracking Control

d	Index denoting the desired (reference) signals
d_{ii}	Damping coefficients ($i = 1,\ldots,3$)
I_{hz}, I_z	Mass moments of inertia (due to hydrodynamic masses and ship mass)
m	Ship mass
m_{ii}	Effective mass parameters ($i = 1,\ldots,3$)
m_{xx}, m_{yy}	Additional hydrodynamic masses
$u_1(t)$	First control input (propeller force)
$u_2(t)$	Second control input (rudder torque, proportional to δ_R)
$v_x(t)$, $v_y(t)$	Velocity components in earth-fixed coordinates

(x, y)	Body-foxed coordinates
$(\tilde{x}, \tilde{y})$	Earth-fixed coordinates
$\delta_{\mathrm{R}}(t)$	Rudder angle
$\omega(t)$	Angular velocity around the center of gravity
$\psi(t)$	Angle of orientation