

David Dahl

**Electromagnetic Modeling and  
Optimization of Through  
Silicon Vias**

# Electromagnetic Modeling and Optimization of Through Silicon Vias

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## Summary

This thesis presents work in the area of electromagnetic modeling of through silicon vias (TSVs). TSVs are vertical interconnects in silicon wafers and an important component for the three-dimensional integration that is required for the further increase in performance of integrated circuits and integrated systems.

Major parts of this thesis discuss the adaptation of the physics-based modeling approach from the original application for the modeling of vias in printed circuit boards to the application in interposers with a sufficient amount of metallizations of the substrate. Adaptations are necessary because the substrate can show significant conductivity and has to be regarded as a layered medium. The latter is due to the required dielectric insulators and due to oxide layers that results from the TSV fabrication process. Further layers need to be included in the analysis if the depletion layer effects due to the metal-oxide-insulator interface are to be considered.

The adaptations consist in the adaptation of a far field model for which an effective wave number of radial wave propagation in the layered medium is computed. They consist also in the computation of a near field model for the mode conversion at the junction between coaxial ports at the top and bottom of the interposer and the inner radial ports that connect to the far field model. In conjunction, the adaptations lead to an efficient and exact modeling over a large parameter range.

The efficient modeling is further applied to large scale crosstalk analysis. A measure for the effective total crosstalk of uncorrelated signal alongs the channels for single-ended links is defined and analyzed for several parameter variations. This measure allows for the investigation of the influence of several important design parameters of silicon interposers on the crosstalk.

Several test structures with TSVs have been fabricated and measured. Using full-wave simulations, the measurement results have been validated.



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# Notation, Symbols and Acronyms

## Notation

$a$	Scalar
$\mathbf{r}$	Vector
$\hat{\mathbf{r}}$	Unit vector in direction of $\mathbf{r}$
$\mathbf{e}_\xi$	Unit vector along coordinate $\xi$
$\mathbf{M}$	Matrix

## Symbols

$\mathbf{1}$	Identity matrix
$\alpha$	Attenuation constant (Np/m)
$\beta$	Phase constant (rad/m)
$C$	Capacitance (F)
$E$	Electric field strength (V m <sup>-1</sup> )
$\varepsilon$	Permittivity
$\underline{\varepsilon}$	Complex permittivity ( $\underline{\varepsilon} = \varepsilon_r \varepsilon_0 - j\sigma/\omega$ )
$\varepsilon_r$	Relative permittivity
$\varepsilon_0$	Permittivity of free space ( $\varepsilon_0 \approx 8.854\,188 \times 10^{-12}$ F m <sup>-1</sup> )
$\eta$	Wave impedance ( $\Omega$ )
$G$	Conductance (S)
$\gamma$	Propagation constant (m <sup>-1</sup> )
$H$	Magnetic field strength (A m <sup>-1</sup> )
$H_n^{(1)}(\cdot)$	Hankel function of the first kind and order $n$
$H_n^{(2)}(\cdot)$	Hankel function of the second kind and order $n$
$I$	Electric current (A)
$\Im$	Imaginary part
$j$	Imaginary unit
$\mathbf{J}$	Matrix of ones
$k$	Complex wave number (m <sup>-1</sup> )

$L$	Inductance (H)
$\mu_r$	Relative permeability of a medium
$\mu_0$	Permeability of free space ( $\mu_0 = 4\pi \times 10^{-7} \text{ H m}^{-1}$ )
$\nabla$	Nabla operator
$\omega$	Angular frequency ( $\text{rad s}^{-1}$ )
$R$	Resistance ( $\Omega$ )
$\Re$	Real part
$S_{n,o}$	Scattering parameter from port $o$ to port $n$ (normalized to $50 \Omega$ if not stated otherwise)
$\sigma$	Electrical conductivity (S/m)
$\sigma_{\text{SD}}$	Standard deviation
$\tan \delta$	Dielectric loss tangent
$V$	Electric voltage (V)
$Y$	Admittance (S)
$Y_{n,o}$	Admittance parameter from port $o$ to port $n$
$Z$	Impedance ( $\Omega$ )
$Z_{n,o}$	Impedance parameter from port $o$ to port $n$
$\mathbb{Z}$	Set of all integer numbers

## Acronyms

2D	two-dimensional
3D	three-dimensional
AC	alternating current
APM	argument principle method
BCB	benzocyclobutene
BOR	body of revolution
CIM	contour integral method
CPU	central processing unit
CTE	coefficient of thermal expansion
CVD	chemical vapor deposition
DUT	device under test
FDFD	finite difference frequency domain
FEM	finite element method
FEXT	far-end cross talk

FFT	fast Fourier transform
FWHM	full width at half maximum
GND	ground
GPIB	General Purpose Interface Bus
GSG	ground–signal–ground
IC	integrated circuit
ISI	inter symbol interference
ITRS	International Technology Roadmap for Semiconductors [1]
LRM	line–reflect–match (calibration)
MEM	multipole expansion
MEMS	micro–electro–mechanical systems
MOS	metal–oxide–semiconductor
MTL	multi–conductor transmission lines
MtM	more–than–Moore
NEXT	near end cross talk
PC	personal computer
PCB	printed circuit board
PDF	probability density function
PEC	perfect electrically conducting
PI	power integrity
PMC	perfect magnetically conducting
PMF	probability mass function
PRBS	pseudo random bit sequence
PSXT	power of cross talk
PUL	per–unit–length
RAM	random–access memory
RDL	redistribution layer
SACVD	sub-atmospheric chemical vapor deposition
SI	signal integrity
Si	silicon
SiO <sub>2</sub>	silicon dioxide
SiP	system in package
SIW	substrate integrated waveguide
SoP	system on package
TEM	transverse electromagnetic
TM	transverse magnetic
TRM	transverse resonance method
TSV	through silicon via

TWA	thin wire approximation ( $\rightarrow$ Section 7.1.4)
UI	unit interval
USB	universal serial bus
via	vertical interconnect access
VNA	vector network analyzer
WPR	wafer photo resist (used as trade name, cf. Fig. 7.9)
WPSXT	weighted power sum of crosstalk
XTALK	crosstalk