

Simon Delchambre

Linear Momentum Enhancement Beta-Factor Estimation Technique for Asteroid Deflection Missions



**Schätzverfahren des Linearen
Beta-Impulsverstärkungsfaktors für
Asteroiden-Ablenkungsmissionen**

Linear Momentum Enhancement Beta-Factor Estimation
Technique for Asteroid Deflection Missions

Simon Delchambre

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Vorsitzender: Prof. Dr.-Ing. Dirk Plettemeier

Gutachter: Prof. Dr. techn. Klaus Janschek
Prof. Dr. A. Fitzsimmons

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Simon Delchambre

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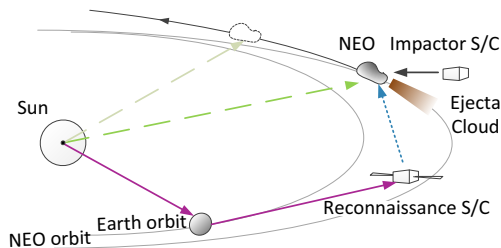
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[Problembeschreibung] Missionen, die die Ablenkung eines Near-Earth Object (NEO) durch einen kinetischen Impaktor demonstrieren, stoßen in der *Planetary Defense Community* auf steigendes Interesse, um die Herausforderungen und die Dynamik von Einschlagszenarien besser verstehen zu können. Von besonderem Interesse ist hierbei der β -Faktor, der die Impulsverstärkung beschreibt, die durch die Energie der ausgeworfenen Materiewolke nach Einschlag entsteht. Die Prädiktion dieses β -Faktors wurde in der Lite-ratur im letzten Jahrzehnt ausführlich behandelt, die Schätzverfahren dazu wurden bisher jedoch nicht untersucht. **[Methodik]** Zunächst liefert diese Arbeit zwei Definitionen des eindeutigen β -Faktors, die beide die Gleichung des Erhaltungssatzes für den linearen Impuls sowie den Drehimpuls lösen. Um den Impulsverstärkungsfaktors genauer schätzen zu können, wird im zweiten Teil der Arbeit eine robuste Beobachtungsstrategie entwickelt. Im dritten Schritt wird mithilfe des Aufklärungssatelliten S/C ein Schätzverfahren zur Bestimmung der heliozentrischen Bahn kleiner NEOs entwickelt, bei dem Radio Science Ortungsdaten mit der Information aus der optischen Navigation fusioniert werden. Abschließend werden die Ergebnisse der Bahnbestimmung zur Schätzung des β -Faktors und des Fehlerbudgets herangezogen.

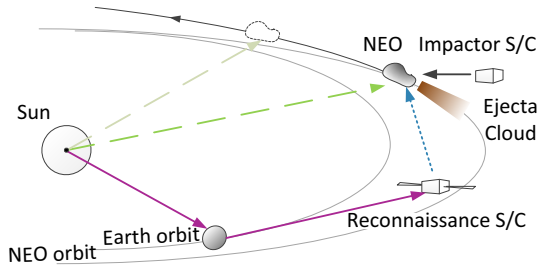


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Abstract

[Background and Problem] A near-Earth Object (NEO) kinetic impactor demonstration mission is gaining interest among the planetary defense community to better understand the challenges and the dynamics of an impact scenario. Of particular interest is the β -factor that expresses the momentum gain caused by ejecta cloud dynamics after impact. Prediction of the β -factor has been addressed extensively in the literature over the last decade, estimation techniques have not been investigated so far.

[Methods] First, this work presents two unambiguous β -factor definitions for the linear and angular momentum conservation equation. Second, a robust observation strategy is developed for the Reconnaissance S/C at the close proximity of the NEO to support the momentum enhancement factor estimation. Third, a NEO heliocentric orbit determination estimator for small NEOs is setup with the support of a Reconnaissance S/C where radioscience tracking data is fused with optical navigation information. Last, the orbit determination results are used to estimate the β -factor and to establish an error budget.



[Results] The estimation performance largely depends on the relative navigation accuracies and the ground station availabilities. Numerical evaluation confirms for the reference scenario a β -factor estimation accuracy of 2% (1σ) after 14 days of monitoring with 67% availability of the ESA tracking stations and one Reconnaissance S/C station-keeping manoeuvre per day. Extending the monitoring window to 28 days only improves the estimation accuracy fractionally. **[Conclusions]** This estimation performance considerably reduces the β -factor uncertainty in a very short time frame and is robust to large uncertainties in the ejecta cloud momentum gain and direction.

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Nomenclature

Acronyms

AIDA	Asteroid Impact and Deflection Assessment
AIM	Asteroid Impact Mission
AOCS	Attitude and Orbital Control Systems
BD	Blast Deflection
CEKF	Consider Extended Kalman Filter
CKF	Consider Kalman Filter
CoM	Center of Mass
DART	Double Asteroid Redirect Test
DCM	Direct Cosine Matrix
Delta-	Delta-Differential One-way Ranging
DOR	
EC	European Commission
EKF	Extended Kalman Filter
ESA	European Space Agency
ESOC	European Space Operations Centre
FIS	Fixed Interval Smoother
GNC	Guidance, Navigation and Control
ISAS	Institute of Space and Astronautical Science
JAXA	Japan Aerospace Exploration Agency
JPL	Jet Propulsion Laboratory
KI	Kinetic Impactor
NASA	National Aeronautics and Space Administration
NEA	Near-Earth Asteroid
NEC	Near-Earth Comet
NEES	Normalized Estimation Error Squared
NEO	Near-Earth Object
NRC	National Research Council
PDF	Probability Density Function
PHA	Potentially Hazardous Asteroid
RSE	Radio Science Experiment
RTS	Rauch, Tung, and Striebel Smoother
S/C	Spacecraft
SKM	Station Keeping Manoeuvre
SPA	Solar Phase Angle
SPG	Stereo-photogrammetry
SRIF	Square Root Information Filter

SRP	Solar Radiation Pressure
SSB	Solar System Barycenter
TRL	Technology Readiness Level

Vector notation

$$\begin{aligned}
 {}^{[\text{ref}]} \mathbf{r}_{[\text{to}]}^{[\text{from}]} &= \text{Position vector from [from] to [to],} \\
 &\quad \text{expressed in reference frame [ref].} \\
 &= [{}^{[\text{ref}]} \mathbf{r}_{[\text{to}]}^{[\text{from}]}(x) \quad {}^{[\text{ref}]} \mathbf{r}_{[\text{to}]}^{[\text{from}]}(y) \quad {}^{[\text{ref}]} \mathbf{r}_{[\text{to}]}^{[\text{from}]}(z)] \quad (1)
 \end{aligned}$$

If [from] would be the origin of the reference frame, it will be omitted as: ${}^{[\text{ref}]} \mathbf{r}_{[\text{to}]}$. If [ref] is a latin letter (e.g. J , it refers to a 3D-reference system. In case of a greek letter (e.g. π), a 3D-projection onto the 2D-plane is meant.

Latin Symbols

Symbol	Description	Units	Dim.
a	Semimajor axis	m	1
A_{sc}	Spacecraft cross surface exposed to solar radiation	m^2	1
B_{sc}	Spacecraft ballistic coefficient	kg m^2	1
c	Speed of light	m/s	1
c_{Rs}^i	Specular reflection coeff. of surface A_i	-	1
c_{Rd}^i	Diffusive reflection coeff. of surface A_i	-	1
c_{Ra}^i	Absorption coefficient of coeff. surface A_i	-	1
$\mathbf{e}_i$	Unit vector in the direction i	-	3×1
$\mathbf{e}$	Residual error between predicted and true measurement i	-	$\text{m} \times 1$
E	Energy	J	1
Ek	Kinetic energy	J	1
Ei	Interna energy	J	1

Ec	Communion energy	J	1
Eh	Internal heat energy	J	1
Em	Elastic wave and other internal energies	J	1
f_x	Probability density function i	-	1
J_{ls}	Least square cost function	dep.	1
J_{mv}	Minimum variance cost function	dep.	1
J_{wls}	Weighted least square cost function	dep.	1
H	Measurement matrix	dep.	$m \times n$
${}^{\pi}_{sc}K$	Intrinsic camera matrix		3×4
n	State size (index)	-	-
m	Measurement size (index)	-	-
m_{imp}	Impactor S/C Mass	kg	1
m_{sc}	Reconnaissance S/C mass	kg	1
M	NEO Mass	kg	1
$J_{\mathbf{n}_i}$	Normal of surface A_i expressed in frame $\{J\}$	-	3×1
$J_{\mathbf{p}_{ej}}$	Ejecta momentum expressed in frame $\{J\}$	kg/m/s	3×1
$J_{\mathbf{p}_{sc}^{neo}}$	Spacecraft pose relative to the NEO	$3 \times m$	7×1
		$4 \times [-]$	
$P_{\odot}$	Solar Constant	$\mu N/m^2$	1
P	Covariance Matrix	dep.	$n \times n$
P^-	A priori state covariance matrix	dep.	$n \times 1$
P^+	A posteriori covariance matrix	dep.	$n \times 1$
${}^{[ref]}_{[to]} \mathbf{r}_{[from]}$	Position vector from [from] to [to], expressed in reference frame [ref]	m	3×1
R	Measurement noise matrix	dep	$m \times m$
$R_{\beta, lin}$	Linear momentum enhancement DCM	-	3×3
$R_{\beta, ang}$	Angular momentum enhancement DCM	-	3×3
$J_{\mathbf{s}_i}$	Sun direction expressed in frame $\{J\}$	-	3×1
$J_{\mathbf{U}}$	Impactor S/C relative velocity expressed in frame $\{J\}$	m/s	3×1
Q	Process noise matrix	dep.	$n \times n$
${}^{\pi}_u$	Feature u-coordinate in image plane $\{\pi\}$	pixel	1
${}^{\pi}_v$	Feature v-coordinate in image plane $\{\pi\}$	pixel	1
$\mathbf{v}$	Process noise	dep.	$n \times 1$
$\mathbf{w}$	Measurement noise	dep.	$m \times 1$
$\mathbf{x}$	True State	dep.	$n \times 1$
$\hat{\mathbf{x}}^-$	Priori state estimate	dep.	$n \times 1$
$\hat{\mathbf{x}}^+$	Posteriori state estimate	dep.	$n \times 1$

Nomenclature

$\tilde{\mathbf{x}}^-$	Priori state estimate error	dep.	$n \times 1$
$\tilde{\mathbf{x}}^+$	Posteriori state estimate error	dep.	$n \times 1$
$\bar{\mathbf{x}}(t)$	Nominal state	dep.	$n \times 1$
X_{neo}	NEO Landmark x-coordinate	m	3×1
$\mathbf{y}$	Theoretical measurements	-	$m \times 1$
$\tilde{\mathbf{y}}$	True (noisy) measurements	dep.	$m \times 1$
$\hat{\mathbf{y}}$	Estimated/Predicted measurement	dep.	$m \times 1$

Greek Symbols

Symbol	Description	Units	Dim.
β	Linear enhancement factor	dep.	1
β_{hh}	Linear enhancement factor, defined by Housen and Holsapple	-	1
β_{sc}	Linear enhancement factor, defined by Scheeres	-	1
β_{lin}	Linear enhancement factor	-	3×3
β_{ang}	Angular enhancement factor	-	3×3
β_{lin}	Linear enhancement gain	-	1
β_{ang}	Angular enhancement gain	-	1
Δ	Difference, e.g. Δv change in velocity	dep.	dep.
ϵ_k	Normalized Estimation Error Squared	-	1
Γ	The input-to-state transition matrix	dep.	dep.
Υ	The noise-to-state transition matrix	dep.	dep.
Φ	The state transition matrix	dep.	dep.
π	Camera plane	n.a.	n.a.
Ψ	Plane defined by Impactor momentum direction and the lever arm w.r.t NEO CoM	n.a.	n.a.
ρ	Correlation coefficient	-	1
ρ_{sc}	Reconnaissance S/C Range Measurement	m	1
$ \rho_{sc} $	Reconnaissance S/C Doppler Measurement	m/s	1
$^J\lambda_{sc}$	Reconnaissance S/C Azimuth Measurement	rad	1
$^J\epsilon_{neo}$	Reconnaissance S/C Elevation Measurement	rad	1
ρ_{neo}	Pseudo NEO Range measurement	m	1

$ \rho_{neo} $	Pseudo NEO Doppler measurement	m/s	1
$J_{\epsilon_{sc}}$	Pseudo NEO Azimuth measurement	rad	1
$J_{\epsilon_{neo}}$	Pseudo NEO Elevation measurement	rad	1
σ	Standard deviation of Gaussian distr.	dep.	1
σ^2	Variation of Gaussian distribution	dep.	1
μ_X	Mean of Gaussian distribution X	dep.	1
μ	Gravitational parameter	m ³ /s ²	1

Math Operators

Symbol	Description
$\cdot$	Dot product
$E\{\}$	Expected value operator
T	Transpose operator
Tr	Trace operator
∇	Gradient
$\times$	Cross-product