

Schriftenreihe des Instituts für Massivbau

Tomás Arana Villafán

## **Fatigue of the Tension-Stiffening Effect in Reinforced Concrete**

Herausgeber:

Prof. Dr. sc. techn. Viktor Sigrist, Prof. Dr.-Ing. Günter Rombach

---

# FATIGUE OF THE TENSION-STIFFENING EFFECT IN REINFORCED CONCRETE

**Vom Promotionsausschuss der  
Technischen Universität Hamburg**

zur Erlangung des akademischen Grades  
Doktor-Ingenieur (Dr.-Ing.)

genehmigte Dissertation

von  
Tomás Arana Villafán

aus  
Sucre

2021

1. Gutachter: Prof. Dr. sc. techn. Viktor Sigrist
2. Gutachter: Prof. Dr.-Ing. Uwe Starossek

Tag der mündlichen Prüfung: 09.04.2021

Schriftenreihe des Instituts für Massivbau der TUHH

Heft 19

**Tomás Arana Villafán**

**Fatigue of the Tension-Stiffening Effect  
in Reinforced Concrete**

Shaker Verlag  
Düren 2021

**Bibliographic information published by the Deutsche Nationalbibliothek**

The Deutsche Nationalbibliothek lists this publication in the Deutsche Nationalbibliografie; detailed bibliographic data are available in the Internet at <http://dnb.d-nb.de>.

Zugl.: Hamburg, Techn. Univ., Diss., 2021

Copyright Shaker Verlag 2021

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording or otherwise, without the prior permission of the publishers.

Printed in Germany.

ISBN 978-3-8440-8107-7

ISSN 1865-8407

Shaker Verlag GmbH • Am Langen Graben 15a • 52353 Düren

Phone: 0049/2421/99011-0 • Telefax: 0049/2421/99011-9

Internet: [www.shaker.de](http://www.shaker.de) • e-mail: [info@shaker.de](mailto:info@shaker.de)

---

## Foreword

---

The computational treatment of the bond between concrete and embedded reinforcing steel bars continues to be the subject of a debate among experts, even after many years of research. This is due to the complexity of the problem, as well as to the further development of construction methods and materials. In addition, with the focus on the assessment of existing structures, the expectations regarding the accuracy of predictions have also changed.

The dissertation presented by Tomás Arana Villafán deals with the bond action under fatigue loading and the influence of its degradation on the stiffness and the deformation capacity of tension chords and the shear resistance of girder webs. The investigations are based on a carefully conducted literature study as well as on tests on tensile elements under fatigue loading carried out at the TUHH. The Tension Chord Model and the method of Generalized Stress Fields serve as an uniform basis for theoretically addressing the issues.

The work is divided into six chapters: The introduction is followed by an overview on stochastically distributed actions and their computation for the example of (large) offshore structures. After that, a comprehensive discussion of the material properties of steel and concrete as well as of the bond behavior under static and repeated loading is presented. The main question of the thesis is pursued in the subsequent chapters. These start with the report on the tests with orthogonally reinforced tension members. Based on the results, the Tension Chord Model is extended for loading and unloading processes by adapting the corresponding design values and by introducing a linear damage function. Exemplarily, the findings are applied to the shear resistance of girder webs; the calculations are compared with experiments from the literature and good agreement is achieved. The work ends with a summary and conclusions.

This dissertation is an important contribution to the knowledge on the fatigue behavior of reinforced concrete structures. Tomás Arana Villafán critically reviews the assumptions made and the results found; in doing so, he points out deficiencies that still exist. Overall, he achieves results of high value for science and engineering practice. These represent a helpful basis for further research.

Lucerne (Switzerland), 2021  
Prof. Dr. Viktor Sigrist



---

## Summary

---

The deformation and carrying capacity of concrete structures depends on the existing bond between concrete and reinforcement. Due to the action of fatigue loads, the bond experiences a degradation process which subsequently modifies the mechanical behaviour of reinforced concrete. In order to quantify the effects of a bond fatigue, diverse tests on concrete chords were performed and evaluated in the present work. A main focus lay on the application of realistic random loads, derived from various sea spectra, since the load history has proven to decisively influence the fatigue behaviour of concrete and steel. Therefore, fatigue tests should reflect the totality of expected loads on a structural element. Additionally, the biaxial stress state around reinforcing bars was systematically varied with the aim of measuring the impact of transverse tension on bond.

The tests results reveal a progressive deterioration of the bond. In consequence, the axial stiffness of tension chords is reduced while the value of crack width after loading increases and the remaining crack width after unloading decreases. A transverse tension does not affect the response behaviour of the tested specimens. Also the ultimate carrying capacity is not negatively altered. However, a higher structural deformation capacity, caused by a weakening of the tension-stiffening effect, could be observed.

In addition, a sort of plastic-strain-accumulation effect in reinforcing bars could be registered. Although the applied loads did not exceed the yield strength  $f_{sy}$  of steel, the reinforcement in most of the tested specimens showed continuously growing plastic deformations. After discussing possible causes, a linear equation is proposed for a quantification of this effect. Further research is required in order to either confirm or refute the development of plastic strains in reinforcement under fatigue loading.

In a further step the nonlinear development of bond degradation is linearised and the Tension Chord Model [115] for static-monotonic loading modified for a mathematical description of the fatigue process. In the proposed model, the rigid-plastic character of bond stresses is kept. The value of bond stresses at serviceability level is linearly reduced depending on the experienced number of load cycles. The reduction is done following the decrease ratio of the tension-stiffening.

Based on the developed model, also a modification of the Generalised Stress Field Analysis [177] is proposed in order to quantify the inclination  $\Theta_{fat}$  of compression struts in concrete beams with web reinforcement under fatigue loading. Results of tests published in the scientific literature reveal a significantly flatter inclination of  $\Theta_{fat}$  as stipulated in design standards, which base on linear stress field analysis. The proposed modification delivers more accurate values of  $\Theta_{fat}$  and enables a more favourable design of beams elements under fatigue.





---

# Contents

---

<b>Preface</b>	<b>c</b>
<b>Summary</b>	<b>e</b>
<b>Acknowledgements</b>	<b>iii</b>
<b>Notation</b>	<b>v</b>
<b>1 Introduction</b>	<b>1</b>
1.1 Context . . . . .	1
1.2 Objective and Overview . . . . .	2
<b>2 Analysis of Random Loads and Stresses for the Fatigue Check</b>	<b>5</b>
2.1 Load Combinations and Required Reliability Index . . . . .	5
2.2 Structural Response . . . . .	10
<b>3 Material Behaviour</b>	<b>33</b>
3.1 Reinforcement . . . . .	33
3.1.1 Reinforcement Behaviour under Static Loading . . . . .	33
3.1.2 Reinforcement Behaviour under Fatigue Loading . . . . .	38
3.2 Concrete . . . . .	49
3.2.1 Concrete Uniaxial Compression Behaviour under Static Loading . . . . .	49
3.2.2 Concrete Uniaxial Tension Behaviour under Static Loading . . . . .	52
3.2.3 Concrete Multiaxial Behaviour under Static Loading . . . . .	54
3.2.4 Concrete Uniaxial Behaviour under Fatigue Compression Loading . . . . .	61
3.2.5 Concrete Uniaxial Behaviour under Fatigue Tension Loading . . . . .	75
3.2.6 Concrete Multiaxial Behaviour under Fatigue Loading . . . . .	77
3.3 Bond . . . . .	79
3.3.1 Bond Behaviour under Static Loading . . . . .	79
3.3.2 Bond Behaviour under Fatigue Loading . . . . .	89
3.3.3 Tension Chord Modell . . . . .	97
<b>4 Tension-Stiffening under Fatigue Loading</b>	<b>99</b>
4.1 Tension-Stiffening Effect in Reinforced Concrete . . . . .	99
4.2 Tension-Stiffening Effect Modelled with the Tension Chord Model . . . . .	107
4.2.1 Experimental Results . . . . .	107

---

4.2.2 Tension Chord Model for Static Loading . . . . .	126
4.2.3 Effective Concrete Area $A_{c,ef}$ . . . . .	135
4.2.4 Fatigue Tension Chord Model . . . . .	138
<b>5 Fatigue Effects on the Structural Behaviour of Reinforced-Concrete Beams</b>	<b>153</b>
5.1 Static Behaviour of Beams in Shear . . . . .	153
5.2 Fatigue Behaviour of Beams in Shear . . . . .	161
<b>6 Summary and Conclusions</b>	<b>169</b>
<b>Bibliography</b>	<b>173</b>
<b>Appendix</b>	<b>195</b>
<b>A Rainflow algorithm</b>	<b>195</b>
<b>B Numerical implementation of the bond-slip-relationship</b>	<b>201</b>
<b>C Numerical implementation of the fatigue Tension Chord Model</b>	<b>211</b>

---

## Acknowledgements

---

Following doctoral thesis was developed mainly during my stay at the Institute of Concrete Structures at the Hamburg University of Technology. I wish to express my deepest gratitude to my supervisor, Prof. Dr. Viktor Sigrist, for the confidence shown to me and for his generous support in every step of this work. I also would like to pay my special regards to Prof. Dr.-Ing. Uwe Starossek for accepting to be the second examiner of this work and for permitting me to use the facilities of the Structural Analysis Institute for the realisation of the fatigue tests. Prof. Dr.-Ing. Rombach supported me repeatedly with valuable advice – I wish to show him my special gratitude. I would like to emphasize the invaluable assistance given by the staff of the Institute of Concrete Structures and of the Structural Analysis Institute: Harald Finger helped me solve numerous practical problems with the test configuration, Stefan Palm-Ziesenitz contributed considerably in the field of metrology, Axel Seils and Olaf Wittleben made the execution of the fatigue tests possible. I also thank Prof. Dr.-Ing. Hintze and his co-workers for facilitating the milling of longitudinal grooves on the reinforcement. I am deeply indebted to my wife and my children: Their infinite patience permitted me culminate this work. I deeply thank you.

Dresden, 2021  
Tomás Arana Villafán



---

## Notation

---

### Roman capital letters

$A_1$	upper limit of the ferrite / cementite phase field
$A_{c,ef}$	effective concrete area
$A_{ci}$	idealised concrete area
$A_{cn}$	net concrete area
$A_{c,red}$	reduced cross sectional area
$A_R$	projected area of single rib
$A_s$	bar cross sectional area
$A_{s,fat}$	effective bar cross sectional area
$A_{sz}$	cross sectional area of reinforcement in z-direction
$\underline{B}$	damping matrix
$C$	constant
$C_1$	parameter
$C_a$	added mass coefficient
$CC$	consequence class
$C_D$	drag coefficient
$C_{Ds}$	drag coefficient for stationary flows
$C_m$	inertia coefficient
$D$	diameter, dimensionless damage
$D_{equ}$	real load-induced damage
$D_{koll}$	damage induced by $\Delta\sigma_{s,equ}$
$EA$	normal stiffness
$E_{agg}$	aggregate modulus of elasticity
$E_c$	concrete modulus of elasticity
$E_{c,fat}$	concrete modulus of elasticity under fatigue loading
$E_{cm}$	concrete secant modulus of elasticity
$E_{c0m}$	concrete tangent modulus of elasticity
$E_i$	idealised modulus of elasticity of uncracked concrete chord
$E_{sm}$	effective modulus of elasticity of tension chord
$E_{sm0}$	effective modulus of elasticity of tension chord at load beginning
$F$	force
$\underline{F}$	force matrix
$F_{cr}$	crack-inducing force
$F_{hyd}$	hydrodynamic force
$F_{ins}$	instationary force

$F_{xV}$	force component of stress field in x-direction
$G$	general failure function
$G_f$	dissipated energy per unit area
$H$	wave height, transfer function
HCF	high cycle fatigue
$H_s$	significant wave height
$K$	wave number, stress concentration range
$\underline{K}$	stiffness matrix
KC	Keulegan-Carpenter number
LCM	low cycle fatigue
$M$	bending moment
$\underline{M}$	mass matrix
$N$	normal force, number of loads
$N_0$	normal force range
$N_f$	total number of load cycles until failure
$N_u$	number of loads cycles that leads to fatigue failure
$N^*$	number of load cycles where inclination of Wöhler curve changes
$P_m$	post-tensioning force
$Q_k$	characteristic value of variable load action
$Q_{k;0.98}$	98% quantile of characteristic value of variable load action
QTF	quadratic transfer function
$R$	range, normal density function of resistance, response spectrum
RAO	response amplification factor
Re	Reynolds number
RC	reliability class
$S$	normal density function of load action, wave spectrum
$S_{c,a}$	relative amplitude compression strength
$S_{c,m}$	relative average compression strength
$S_{c,max}$	relative maximum compression strength
$S_{c,min}$	relative minimum compression strength
$S_S$	response spectrum
$S_1$	sea spectrum
$T$	period
$T_c$	mean wave period
$T_{cyc}$	load period
$T_{mg}$	melting temperature of steel
$T_t$	transition temperature which leads to creep in steel
$T_p$	peak wave period
$T_z$	zero-up crossing period of wave
$T_{z,\alpha}$	zero-up crossing period of response
$U_{cF}$	specific fracture energy
$V$	shear force
$V_{fat}$	shear force under fatigue loading
$V_{R,c}$	shear resistance capacity of web concrete

$V_{R,sy}$	shear resistance capacity of web reinforcement
$Y$	yield function
$Y_c$	yield function of plain concrete
$Y_{c1}$	first yield limit of plain concrete
$Y_{c2}$	second yield limit of plain concrete
$Y_s$	response transfer function
Roman lower case letters	
$a$	water acceleration, crack length
$a_0, a_1, \dots, a_n$	Fourier coefficients, parameters
$\hat{a}$	amplitude
$a_{sx}$	lengthwise cross sectional area of reinforcement in x-direction
$a_{sw}$	lengthwise cross sectional area of web reinforcement
$a_{sz}$	lengthwise cross sectional area of reinforcement in z-direction
$b$	damping coefficient
$b_0, b_1, \dots, b_n$	Fourier coefficients
$b_{ffi_s}$	parameter
$b_\emptyset$	parameter
$b_w$	web width
$c_0, c_1$	constant values
$c_{nom}$	concrete cover
$c_s$	internal concrete cohesion
$d$	static height
$d_w$	water depth
$f$	frequency
$f_c$	uniaxial concrete compression strength
$f_{c,cube}$	uniaxial concrete compression strength tested on a cube
$f_{ce}$	concrete effective compression strength
$f_{c,fat}$	uniaxial fatigue strength of concrete
$f_{ct}$	concrete uniaxial tension strength
$f_{ct;0.05}$	5%-quantile of concrete uniaxial tension strength
$f_{ct;0.95}$	95%-quantile of concrete uniaxial tension strength
$f_{load}$	load frequency
$f_R$	bond index
$f_{su}$	steel uniaxial ultimate strength
$f_{sy}$	steel uniaxial yield strength
$f_t$	uniaxial tension strength
$g$	gravity constant
$k$	coefficient of $f_{py}/f_{sy}$ , inclination of Wöhler curve
$k_t$	reduction factor
$k_\emptyset$	factor for quantification of $\tau_{bU}$ and $\tau_{bR}$ in dependence of $f_{ct}$
$k_u$	displacement coefficient, coefficient of steel stress at cracked section by reloading and at first cracking



$l_b$	bond length
$l_{by}$	transmission length by yielding of reinforcement
$m$	exponent for Wöhler curve
$m_a$	added mass
$m_0, m_2, m_4$	statistical moments
$m_{pl}$	slope of $\varepsilon_{sm,pl} - n/N$ -curve
$m_{sm}$	slope of $E_{sm} - n/N$ -curve
$m_{yy}$	lengthwise bending moment
$n$	natural number, coefficient of $E_s/E_c$
$n_{equ}$	equivalent number of load cycles
$n_x$	axial membrane forces in x-direction
$n_z$	axial membrane forces in z-direction
$\varnothing_s$	reinforcing bar diameter
$p$	pressure, probability
$p_0$	atmospheric pressure
$p_f$	failure probability
$p_{ins}$	instationary pressure
$p_r$	radial compression
$r_1, r_2$	parameters of meridians
$r_c$	radius function
$r_i$	radius of inner concrete ring
$r_o$	radius of outer concrete ring
$s_{rm}$	average crack spacing
$s_{rm,max}$	maximum average crack spacing
$s_{rm,min}$	minimum average crack spacing
$s_{r0}$	maximum distance between cracks
$u$	mode value, velocity in x-direction
$u_1$	mode value in a reference period of 1 year
$u_{50}$	mode value in a reference period of 50 years
$u_a$	amplitude velocity of oscillating fluid
$v$	velocity in y-direction
$w$	velocity in z-direction
$w_{cr}$	crack width
$w_{cr,lim}$	limit value of crack width
$w_{tr}$	transverse crack width
$w_u$	crack width by which no force transmission is possible
$x_{sR}$	intersection point of steel stress distribution between unloading and reloading process
$x_{sU}$	intersection point of steel stress distribution between loading and unloading process
$\underline{x}$	deformation or motion matrix
$\underline{\dot{x}}$	velocity matrix
$\underline{\ddot{x}}$	acceleration matrix

$\dot{x}_r$	velocity matrix of rigid body
$\ddot{x}_r$	acceleration matrix of rigid body
$z$	lever arm of internal forces
Greek letters	
$\alpha_1, \alpha_2, \alpha_3$	parameters
$\alpha_b$	inclination angle of stress resultant around reinforcing bar
$\alpha_{c,duc}$	ductility degree factor
$\alpha_e$	$= E_s/E_c$
$\alpha_i$	parameter
$\beta_b$	inclination angle between reinforcing bar and concrete wedge
$\beta_{c,fat}$	$= \varepsilon_{c,fat,da}/\varepsilon_{c,fat}$
$\beta_{RC}$	reliability index
$\beta_i$	mean direction of sea state
$\gamma$	shearing strain
$\gamma_Q$	safety factor for variable load action
$\gamma_{s,fat}$	material safety factor at fatigue limit state
$\gamma_{Sd}$	load safety factor at fatigue limit state
$\gamma_{xz}$	shearing strain in xz-plane
$\Delta\varepsilon_{s,el}$	elastic steel strain range
$\Delta\varepsilon_{s,pl}$	plastic steel strain range
$\Delta\varepsilon_{sm0}$	difference of average steel strain resulting from degradation of tension-stiffening
$\Delta\sigma_s$	normal stress range
$\Delta\sigma_{s0}$	$= N_0/A_s$
$\Delta\sigma_{s,equ}$	damage equivalent stress range
$\delta_d$	factor for steel ductility
$\delta_s$	slip between reinforcing bar and surrounding concrete
$\delta_{s0}$	slip $\delta_s$ under static-monotonic loading
$\delta_{s1}$	slip $\delta_s$ after one load cycle
$\delta_{sr}$	residual slip
$\delta_{s,max}$	maximum bond slip
$\epsilon_1$	principal strain
$\epsilon_3$	principal strain
$\epsilon_c$	concrete strain
$\epsilon_{c1}, \epsilon_{c3}$	concrete principal strains
$\epsilon_{c0}$	uniaxial concrete compression strength at failure
$\epsilon_{c3,el}$	elastic component of concrete strain under fatigue
$\epsilon_{c3,t}$	time-dependent component of concrete strain under fatigue
$\epsilon_{c,fat}$	concrete strain under fatigue loading
$\epsilon_{c,fat,da}$	damage-induced strain under fatigue
$\epsilon_{cm}$	average concrete strain
$\epsilon_{ct}$	$= f_{ct}/E_c$

$\epsilon_{cu}$	concrete failure strain
$\epsilon_{cu,fat}$	concrete ultimate fatigue strain
$\epsilon_n$	phase angle
$\epsilon_r$	remaining strain after unloading
$\epsilon_{sm}$	average steel strain
$\epsilon_{sm,cal}$	calculated average steel strain
$\epsilon_{sm,mes}$	measured average steel strain
$\epsilon_{sm,\emptyset}$	bond-dependent average steel strain
$\epsilon_{sm,pl}$	plastic part of average steel strain
$\epsilon_{s,max}$	maximal steel strain
$\epsilon_{smz}$	average steel strain in z-direction
$\epsilon_{sr}$	steel strain at at midpoint between cracks
$\epsilon_{sr0}$	steel strain at cracked section
$\epsilon_{srx}$	steel strain at cracked section in x-direction
$\epsilon_{srz}$	steel strain at cracked section in z-direction
$\epsilon_{sy}$	uniaxial steel yield strain
$\epsilon_{uf}$	strain of flange of a composite bridge
$\epsilon_x$	strain in x-direction
$\epsilon_z$	strain in z-direction
$\zeta$	water level function
$\zeta_a$	wave amplitude, response amplitude
$\eta_D$	limit of cumulative damage ratio
$\eta_\delta$	parameter for quantification of slip reversal
$\eta_{TS}$	tension-stiffening number
$\Theta$	inclination angle
$\theta$	relative roughness
$\Theta_c$	Haigh-Westergaard-coordinate
$\Theta_{fat}$	inclination of compression strut at fatigue limit state
$\Theta_{fat,cal}$	calculated inclination of compression strut at fatigue limit state
$\Theta_{fat,MC2010}$	inclination of compression strut at fatigue limit state according to Model Code 2010
$\Theta_{fat,mes}$	measured inclination of compression strut at fatigue limit state
$\Theta_{FB,stat}$	inclination of compression strut at ultimate limit state according to DIN Fachbericht 102
$\Theta_{max}$	maximal inclination angle
$\Theta_{min}$	minimal inclination angle
$\Theta_{ult}$	inclination of compression strut at the ultimate limit state
$\Theta_z$	yaw motion
$K_{b0}$	bond strength according to Tension Chord Model
$K_{c,fat}$	damage parameter
$K_{rsd}$	calculation factor of residual bond strength
$K_{t,fat}$	damage parameter
$K_\tau$	bond coefficient

$\lambda$	parameter for quantification of distance between cracks
$\lambda_w$	wave length
$\mu$	mean value
$\mu_c$	elastic Poisson's ratio
$\mu_G$	mean value of $G$
$\mu_{H_s, \sigma_s}$	median value of significant stress in reinforcement
$\mu_R$	mean value of $R$
$\mu_S$	mean value of $S$
$\nu$	Poisson's number
$\xi$	parameter
$\xi_c$	Haigh-Westergaard-coordinate
$\rho_c$	Haigh-Westergaard-coordinate
$\rho_{s,ef}$	effective geometrical reinforcement ratio
$\rho_{sx}$	geometrical reinforcement ratio in x-direction
$\rho_{sz}$	geometrical reinforcement ratio in z-direction
$\rho_w$	water density
$\sigma$	normal stress
$\sigma_1$	principal stress
$\sigma_3$	principal stress
$\sigma_{br}$	radial stress
$\sigma_{b\phi}$	circumferential stress
$\sigma_c$	concrete normal stress
$\sigma_{c1}$	concrete principal stress
$\sigma_{c3}$	concrete principal stress
$\sigma_{c,D}$	real stress due to fatigue-induced damage of cross section
$\sigma_{cx}$	concrete normal stress in x-direction
$\sigma_{cz}$	concrete normal stress in z-direction
$\overline{\sigma_s}$	mean steel stress
$\sigma_{H_s, \sigma_s}$	standard deviation of significant stress in reinforcement
$\sigma_{s,max}$	maximal steel stress
$\sigma_{sr}$	steel stress at cracked section
$\sigma_{sr0}$	steel stress at cracked section immediately after crack formation
$\sigma_{std}$	standard deviation
$\sigma_{sx}$	steel stress in x-direction
$\sigma_{sw}$	steel stress in web reinforcement
$\sigma_{sz}$	steel stress in z-direction
$\sigma_x$	normal stress in x-direction
$\sigma_z$	normal stress in z-direction
$\tau$	shear stress
$\tau_b$	bond strength
$\tau_{bf}$	frictional bond strength
$\tau_{b,0.1}$	bond stress at a slip of 0.1 mm
$\tau_{b0}$	rigid-plastic bond strength for $\sigma_s < f_{sy}$
$\tau_{b1}$	rigid-plastic bond strength for $\sigma_s \geq f_{sy}$

$\tau_{b,max}$	ultimate bond strength
$\tau_{bR}$	rigid-plastic bond stress by reloading
$\tau_{bR1}$	rigid-plastic bond stress by 1 <sup>st</sup> cycle of reloading
$\tau_{bR,rsd}$	residual rigid-plastic bond stress by reloading
$\tau_{bU}$	rigid-plastic bond stress by unloading
$\tau_{bU1}$	rigid-plastic bond stress by 1 <sup>st</sup> cycle of unloading
$\tau_{bU,rsd}$	residual rigid-plastic bond stress by unloading
$\tau_c$	concrete shear stress
$\tau_{cxz}$	concrete shear stress in the xz-plane
$\tau_{R,max}$	ultimate shear stress
$\tau_{xz}$	shear stress in the xz-plane
$\Phi$	potential function
$\Phi_b$	conical shell expansion of bond stresses
$\phi_c$	angle of internal friction of concrete
$\Phi_n$	phase angle
$\Phi_s$	potential function of diffracted wave
$\Phi_w$	potential function of undisturbed wave
$\Phi_x$	roll motion
$\Psi_{KC}$	modification factor for instationary flows
$\Psi_y$	pitch motion
$\psi_0, \psi_1, \psi_2$	combination factor
$\omega$	circular frequency
$\omega_{sy}$	mechanical reinforcement ratio
$\omega_p$	circular peak frequency
$\omega_{py}$	mechanical ratio of prestressing steel
$\omega_T$	wave circular frequency