

Modellbildung und Regelung mechatronischer Systeme

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Ngoc Danh Dang

Advanced Control Designs for Output Tracking of Hydrostatic Transmissions

**Berichte aus dem
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Universität Rostock**

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Rostock, November 2021

Ngoc Danh Dang

Abstract

Hydrostatic transmissions, in comparison to conventional mechanical transmission systems, expose many advantages. They provide continuously variable transmission ratios, high power density, low inertia, efficient operation in a wide range of torque-to-speed ratios and they can serve as a dynamic braking system. The spatial arrangement of a hydrostatic transmission is very flexible also, the power can be transmitted from a single prime mover to several load locations, even in the cases where the position and orientation of the load units change. In industrial applications, hydrostatic transmission systems are widely used in specialized working vehicles such as construction, agriculture, excavation machinery and off-road vehicles where high drive torques are required. In other transport vehicles, passenger cars for instance, the applications of hydrostatic transmissions are less common. Recently, in the trend of emission reduction and environment-friendly applications, the hydrostatic transmissions gain more and more contribution in high-performance transport vehicles with the invention of power-split gearboxes, energy recovery systems in hybrid drive trains and also in green energy systems such as ocean energy systems and wind turbines, etc.

The limitation of hydrostatic transmissions in high-performance applications is caused by the energy efficiency and control issues. From a control point of view, they are characterized by high nonlinearity, disturbance and physical parameter uncertainty caused by many operational and structural aspects such as fluid viscosity, temperature variation, leakage oil flow and the elasticity of the connecting hoses, etc. Currently, PID (proportional-integral-derivative) controllers are still predominant in HST applications but their performance, however, is not sufficient to attain an accurate control result in the wide range of HST operation. Therefore, advanced control approaches become more favorable.

In the last decade, many nonlinear control approaches have been proposed, those are diverse in terms of control strategy, control objective and design principle. Based on the existing control techniques that have been proposed generally for uncertain nonlinear systems, the work in this dissertation focuses on the design and the validation of advanced control approaches for the output tracking of HST systems.

Taking the practical considerations as a guideline, the work addresses simple but efficient model descriptions in a combination with advanced control and estimation approaches to achieve an accurate tracking of the desired trajectories. The proposed control designs are capable of fully exploiting the wide operation range of HSTs within the system configuration limits. The dissertation develops a new trajectory planning scheme for the output tracking of HST systems that efficiently and simultaneously uses both the primary and secondary control inputs. Based on this control scheme, simple design models or even purely data-driven models are envisaged and deployed to develop and investigate several advanced control approaches for HST systems: optimal control, estimator-based feedback linearization control, active disturbance rejection control and model-free control approaches. The use of tracking differentiators – which can be interpreted as a model-free way to determine time derivatives of noise-afflicted measurements and substitute classical state transformations corresponding to a classical model-based approach – is investigated in many of the mentioned control structures. Thereby, a practical view on the applicability of such technical measures for effective and robust control designs on HST systems is provided. Successful study results are obtained by means of both simulations and experiments on a real test rig of the hydrostatic transmission – which is built for validation tests at the Chair of Mechatronics, University of Rostock.

Zusammenfassung

Hydrostatische Getriebe bieten im Vergleich zu herkömmlichen mechanischen Getrieben viele Vorteile. Sie ermöglichen stufenlos verstellbare Übersetzungsverhältnisse, eine hohe Leistungsdichte, ein geringes Trägheitsmoment, einen effizienten Betrieb in einem weiten Bereich von Drehmoment-Drehzahl-Verhältnissen, und sie können als dynamisches Bremssystem dienen. Die räumliche Anordnung eines hydrostatischen Getriebes ist ebenfalls sehr flexibel, die Leistung kann von einer einzigen Antriebsmaschine auf mehrere Laststellen übertragen werden, auch wenn sich die Position und Ausrichtung der Lasteinheiten ändert. In der Industrie werden die hydrostatische Getriebesysteme häufig in speziellen Arbeitsfahrzeugen wie Bau-, Landwirtschafts-, Bagger- und Geländefahrzeugen eingesetzt, die hohe Antriebsmomente erfordern. In anderen Transportfahrzeugen, z.B. in Personenkraftwagen, ist der Einsatz hydrostatischer Getriebe weniger verbreitet. In jüngster Zeit gewinnen hydrostatische Getriebe im Zuge des Trends zur Emissionsreduzierung für leistungsstarke Transportfahrzeuge und umweltfreundlichen Anwendungen mit der Entwicklung von leistungsverzweigten Getrieben, Energierückgewinnungssystemen in Hybridantriebssträngen und auch in umweltfreundlichen Energiesystemen wie Meeresenergiesystemen und Windturbinen immer mehr an Bedeutung.

Die Grenzen des hydrostatischen Getriebes in Hochleistungsanwendungen liegen in der Energieeffizienz und der Regelung. Aus regelungstechnischer Sicht sind sie durch hohe Nichtlinearität, Störungen und Unsicherheiten der physikalischen Parameter gekennzeichnet, die durch zahlreiche betriebliche und strukturelle Aspekte wie Flüssigkeitsviskosität, Temperaturschwankungen, Leckölströme und Elastizitäten der Verbindungsschläuche usw. verursacht werden. Gegenwärtig sind PID-Regler (proportional-integral-differenzierend) in HST-Anwendungen immer noch vorherrschend, aber ihr Leistungsvermögen reicht nicht aus, um in dem weiten Bereich des HST-Betriebs ein genaues Regelungsverhalten zu erzielen. Daher werden fortschrittliche Regelungsansätze immer bedeutender.

Im letzten Jahrzehnt wurden viele nichtlineare Regelungsansätze vorgeschlagen, die sich in Bezug auf Regelungsstrategie, Regelungsziel und Entwurfsprinzip unterscheiden. Basierend auf den bestehenden Entwurfstechniken, die im Allg. für unsichere nichtlineare Systeme vorgeschlagen wurden, konzentriert sich diese Dissertation auf den Entwurf und die Validierung von fortgeschrittenen Regelungsansätzen für die Ausgangsfolgeregelung von HST-Systemen.

Unter Berücksichtigung praktischer Erwägungen werden in dieser Arbeit einfache, aber effiziente Modellbeschreibungen in Kombination mit fortschrittlichen Regelungs- und Schätzmethoden verwendet, um eine genaue Verfolgung der gewünschten Trajektorien zu erreichen. Die vorgeschlagenen Regelungskonzepte sind in der Lage, den weiten Betriebsbereich von HSTs innerhalb der Grenzen der Systemkonfiguration voll auszunutzen. In der Dissertation wird ein neues Schema für die Trajektorienplanung für die Ausgangsfolge von HST-Systemen entwickelt, das sowohl die primären als auch die sekundären Steuereingänge effizient und gleichzeitig nutzt. Auf der Grundlage dieses Strukturierung werden einfache Entwurfsmodelle oder sogar rein datengetriebene Modelle in Betracht gezogen und eingesetzt, um mehrere fortschrittliche Regelungsansätze für HST-Systeme zu entwickeln und zu untersuchen: optimale Regelung, schätzungsbasierte Linearisierungsverfahren im Rückführzweig, aktive Störungsunterdrückung und modellfreie Regelungsansätze. Die Verwendung von Tracking-Differenzierern – die als modellfreier Weg zur Bestimmung von Zeitableitungen störungsbehafteter Messungen interpretiert werden können und klassische Zustandstransformationen, die einem klassischen modellbasierten Ansatz entsprechen, ersetzen – wird in vielen der genannten Regelungsstrukturen untersucht. Dadurch wird ein praktischer Blick auf die Anwendbarkeit solcher technischer Maßnahmen für

effektive und robuste Regelungsentwürfe auf HST-Systemen ermöglicht. Erfolgreiche Untersuchungsergebnisse werden sowohl mittels Simulationen als auch durch Experimente an einem realen Prüfstand des hydrostatischen Getriebes – der für Validierungstests am Lehrstuhl für Mechatronik der Universität Rostock aufgebaut ist – erzielt.

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Abbreviations

ADR	Active disturbance rejection
AFC	Adaptive feedforward compensation
APE	Adaptive parameter estimation
CMAC	Cerebellar model articulation controller
DARE	Discrete algebraic Riccati equation
DLQR	Discrete linear quadratic regulator
ESO	Extended state observer
FBO	Takagi-Sugeno Fuzzy-based optimal control
FEL	Feedback error learning
Fig.	Figure
GPI	Generalized-proportional-integral
HJB	Hamilton-Jacobi-Bellman
HST	Hydrostatic transmission
iPD	Intelligent proportional-derivative control
LQR	Linear quadratic regulator
LTD	Linear tracking differentiator
LTD2	Second-order linear tracking differentiator
LTD3	Third-order linear tracking differentiator
LTD4	Fourth-order linear tracking differentiator
LMI	Linear matrix inequality
MIMO	Multiple input-multiple output
MISO	Multiple input-single output
MLP	Multiple layer perceptron network
MPC	Model predictive control
NMPC	Nonlinear model predictive control
P	Proportional control
PD	Proportional-derivative control
PI	Proportional-integral control
PID	Proportional-integral-derivative control
RDO	Reduced-order state and disturbance observer
RBF	Radial basic function
RMS	Root mean square
SDRE	State-dependent Riccati equation
SIF	State-dependent integral feedback control
SISO	Single input-single output
SMC	Sliding mode control
Tab.	Table
TD	Tracking differentiator
UKF	Unscented Kalman filter
w.r.t.	with respect to

List of Symbols

The list is split into parts according to the structure of the content. Note that supplementary, intermediate, and auxiliary variables are not explicitly listed, but specified at the place of their first usage.

Modeling and Trajectory Planning for the HST System

Mathematical Model of the HST system

q_P	Hydraulic pump flow rate
q_M	Hydraulic motor flow rate
ω_P	Hydraulic pump angular velocity
ω_M	Hydraulic motor angular velocity
V_P	Volumetric displacement of hydraulic pump
V_M	Volumetric displacement of hydraulic motor
$\tilde{V}_P$	Maximal volumetric displacement of hydraulic pump
$\tilde{V}_M$	Maximal volumetric displacement of hydraulic motor
α_P	Swash-plate angle of hydraulic pump
$\alpha_{P,max}$	Maximal swash-plate angle of hydraulic pump
$\tilde{\alpha}_P$	Normalized swash-plate angle of hydraulic pump
α_M	Bent-axis angle of hydraulic motor
$\alpha_{M,max}$	Maximal bent-axis angle of hydraulic motor
$\tilde{\alpha}_M$	Normalized bent-axis angle of hydraulic motor
ϵ_M	Lower bound of hydraulic motor bent-axis angle
A_P	Effective piston area of hydraulic pump
D_P	Piston circle diameter of hydraulic pump
N_P	Number of piston in hydraulic pump displacement unit
A_M	Effective piston area of hydraulic motor
D_M	Piston circle diameter of hydraulic motor
N_M	Number of piston in hydraulic motor displacement unit
Δp	Difference pressure in the hydraulic circuit
q_U	Lumped leakage flow disturbance
τ_U	Lumped torque disturbance
T_{uP}	Time constant of hydraulic pump actuator
T_{uM}	Time constant of hydraulic motor actuator
k_P	Proportional gain of hydraulic pump actuator
k_M	Proportional gain of hydraulic motor actuator
u_P	Analog control signal of hydraulic pump
u_M	Analog control signal of hydraulic motor
J_V	Mass moment of inertial at hydraulic motor shaft

d_V	Damping coefficient at hydraulic motor shaft
$\tilde{\omega}_{Md}$	Desired trajectory of motor bent-axis angle
ω_{Md}	Desired trajectory of motor angular velocity
$\omega_{Md,max}$	Maximal desired value of motor angular velocity
ω_{max}	Physically-limited value of motor angular velocity
a	User-defined threshold for motor displacement activation
b	Limit parameter for desired trajectory design

Advanced Output Control Designs

Optimization-Based Approaches

Model Predictive Control

N	Number of point in prediction horizon
J	Cost function
w_j	Weighting scalars, $j = \{1, 2, 3, 4\}$
k	Time index
p	Design parameter
$\mathbf{x}$	Full system state vector
$\tilde{\gamma}$	Lyapunov function candidate
$\mathbf{f}$	System dynamics vector function
$\mathbf{C}_m$	Output matrix
$\mathbf{x}_e$	Extended system state vector
Ψ	Extended system dynamics vector function
α, κ, β	Design parameters
$\hat{\mathbf{x}}_e$	Estimate of extended system state vector
$\mathbf{Q}_K$	Process noise covariance matrix
$\mathbf{R}_K$	Measurement noise covariance matrix
$\mathbf{K}_K$	Kalman gain matrix
$\mathbf{L}_x$	State covariance matrix
$\tilde{\mathbf{L}}_x$	Error covariance matrix
$\tilde{\mathbf{L}}_y$	Measurement error covariance matrix
$\mathbf{L}_{xy}$	Cross-covariance matrix
τ_h	Motor hydraulic torque
τ_{hd}	Desired value of hydraulic torque

Takagi-Sugeno Fuzzy-Based Optimal Control Design for Torque Tracking Control

$y_\tau = \tau_h$	Output torque
$\mathbf{x}_\tau$	Torque-controlled system state vector
v	Lump disturbance input
$\mathbf{A}$	Quasi-linear system matrix
$\mathbf{b}$	Input matrix
$\mathbf{d}$	Disturbance input matrix
$\mathbf{c}$	Output matrix

$\mathbf{A}_{d,1}, \mathbf{A}_{d,2}$	Discretized vertex system matrices
$\mathbf{b}_d$	Discretized input matrix
$\mathbf{d}_d$	Discretized disturbance input matrix
$\mathbf{K}_C$	Kalman controllability criterion matrix
$\mathbf{A}_1, \mathbf{A}_2$	Vertex system matrices
h_1, h_2	Weighting functions
$\mathbf{S}_1, \mathbf{S}_2$	DARE solutions matrices
$\mathbf{A}_{c,1}, \mathbf{A}_{c,2}$	Vertex closed-loop system matrices
$\mathbf{A}_c$	Closed-loop system matrix
$\mathbf{k}_1, \mathbf{k}_2$	Vertex optimal feedback gain vectors
$\mathbf{k}$	Optimal feedback gain vector
$\mathbf{P}_L$	Common Lyapunov function
J_τ	Quadratic cost function
$\mathbf{Q}$	State weighting matrix
r	Control input weighting scalar
T_s	Sampling time
$k_{F,0}, k_{F,1}, k_{F,2}$	Feedforward gains
$k_{D,0}, k_{D,1}$	Disturbance compensation gains
z	z-transform variable
$y_{\tau,d} = \tau_{hd}$	Desired value of output torque
a_M	System matrix of motor dynamics
b_M	Input vector of motor dynamics system
a_{dM}	Discretized system matrix of motor dynamics
b_{dM}	Discretized input vector of motor dynamics system
k_M	Optimal feedback gain of motor control
b_M	Input vector of motor dynamics system
$k_{MF,0}, k_{MF,1}$	Feedforward gains of motor control

State-Dependent Integral State Feedback Control

$\bar{\mathbf{x}}$	Augmented state vector
$\bar{\mathbf{A}}$	Augmented system matrix
$\bar{\mathbf{b}}$	Augmented input matrix
$\bar{\mathbf{e}}$	Reference input matrix
$\bar{\mathbf{c}}$	Augmented output matrix
w_τ	Integral state
$\bar{\mathbf{K}}_C$	Kalman controllability matrix
$\bar{\mathbf{A}}_d$	Discretized augmented system matrix
$\bar{\mathbf{b}}_d$	Discretized augmented input matrix
$\bar{\mathbf{k}}$	State-dependent feedback gain vector
$\bar{\mathbf{Q}}$	Weighting matrix of augmented state vector
$\bar{r}$	Weighting scalar of feedback control input
u_{SFB}	Feedback control input
$\bar{J}_\tau$	Quadratic cost function
$\bar{\mathbf{S}}$	SDRE solution matrix
$\bar{\mathbf{A}}_c$	Close loop system matrix
$\bar{\mathbf{A}}_{d,i}$	Vertex discretized system matrices, $i = \{1, 2, 3, 4\}$

$\bar{\mathbf{A}}_{c,i}$	Vertex close loop system matrices, $i = \{1, 2, 3, 4\}$
$\bar{\mathbf{K}}_i$	Vertex system feedback gain vectors, $i = \{1, 2, 3, 4\}$
$\bar{\mathbf{P}}_L$	Common Lyapunov function matrix
$\bar{\mathbf{A}}_c$	State feedback close loop system matrix
$k_{V,0}, k_{V,1}$	State-dependent disturbance compensation gains
u_{SDC}	Disturbance compensation control input
J_e	Quadratic cost function for observer design
$\mathbf{K}_e$	Kalman observability criterion matrix
$\mathbf{Q}_e$	Weighting matrix of estimation error
$\mathbf{R}_e$	Weighting matrix of measurement error
$\mathbf{A}_{ed}$	Discretized extended system matrix
$\mathbf{B}_{ed}$	Discretized extended system matrix
$\mathbf{S}_e$	SDRE solution for observer design
$\mathbf{H}_e$	State-dependent observer gain matrix
$\mathbf{H}_{e,i}$	Vertex observer gain matrix, $i = \{1, ..., 8\}$
$\mathbf{O}_{e,i}$	Vertex observer system matrix, $i = \{1, ..., 8\}$
$\mathbf{P}_e$	Common Lyapunov function matrix

Estimator-Based Feedback Linearization

ς_i	State variables of tracking differentiator $i = 1, 2, ..., n$
$c_{\varsigma n,i}$	Tracking differentiator coefficients $i = 1, 2, ...n$
$R_{\varsigma n}$	Tuning parameter for tracking differentiator
r_x	Tracked signal
$\mathbf{y}$	System state vector
V	Lyapunov function candidate
λ_0, λ_1, k	Feedback-linearized control parameters
e	Tracking error
f	Nonlinearity function
$c_{\varsigma 3,i}$	LTD3 coefficients $i = 1, 2, 3$
$R_{\varsigma 3}$	Tuning parameter for LTD3

Nonlinearity Compensation Using a State and Disturbance Observer

$\hat{\mathbf{q}}$	Estimated state vector of RDO
$\mathbf{p}$	Internal state vector of RDO
$\mathbf{H}$	Observer gain matrix of RDO
$\mathbf{y}_m$	Measurement vector
$\mathbf{u}$	Control input vector
ξ	Estimation error vector of RDO
Φ	Nonlinear function of RDO
$\mathbf{J}_\xi$	Jacobian matrix of estimation error dynamics of RDO
$\mathbf{f}_1, \mathbf{f}_2$	System dynamics vector functions
$\mathbf{I}$	Identity matrix
$k_{\alpha M}$	Feedback gain of hydraulic motor control

Nonlinearity Compensation by Adaptive Parameter Estimation

s	Laplace domain variable
$\tilde{f}$	Alternative presentation of nonlinearity function in adaptive control
$\tilde{k}$	Adaptive control design parameter
$\hat{a}_1$	Adaptive parameters, $i = \{0, 1, 2, 3, 4\}$
γ_i	Adaptation rate for parameter estimation, $i = \{0, 1, 2, 3, 4\}$
$y_d = \omega_{Md}$	Desired value of angular velocity

Nonlinearity Compensation by a Neural Network

F_N	Output function of neural network
$\tilde{k}$	Feedback control design parameter
$\tilde{f}$	Alternative presentation of nonlinearity function in neural feedback control
$\mathbf{w}$	Neural network output weight vector
$\mathbf{V}$	Neural network hidden-layer weight matrix
σ	Neural network activation function
$M_w, \mathbf{M}_v, \kappa, \kappa_r$	Neural network tuning parameter
Θ	Augmented weighting matrix of neural network
θ	Bound of augmented weighting matrix norm
Q	Norm bound of desired trajectory
u_δ	Robust term of neural network control
$\mathbf{x}_N$	Neural network synapsis input vector

Active Disturbance Rejection Approaches

Observer-Based ADR Control Design

ζ	Estimation errors of ESO
$\hat{y}$	System states estimated by ESO
χ_1, χ_2	Extended states of ESO
$\tilde{F}$	Total disturbance in ADR control
l_i	Gains of ESO, $i = \{0, 1, 2, 3, 4\}$
α_i	Feedback gains of ADR control, $i = \{0, 1, 2\}$
t, τ	Time variable
P_0	Characteristic polynomial of ESO
P_e	Characteristic polynomial of linear feedback controller

Flat-Filtering ADR Control Design

e_{uP}	Control input error
β_i	Flat-filtering network gains, $i = \{0, 1, 2, 3, 4, 5\}$
P_β	Characteristic polynomial

Model-Free approaches

y	Output of phenomenological model
u	Input of phenomenological model
F	Piece-wise constant function of disturbance in ultra-local model
K	User-defined system parameter

Sliding Mode Control

$\hat{F}$	Estimate of disturbance in ultra-local model
$\tilde{\xi}$	Disturbance estimation error
$\bar{L}$	Disturbance estimation error bound
ϑ	Sliding variable
ρ_0, ρ_1, ρ_2	Sliding manifold design parameters
$\bar{\eta}, \bar{\rho}, \bar{\gamma}$	Sliding control design parameters
$\bar{V}$	Lyapunov function candidate
$R_{\varsigma 4}$	LTD4 parameter
$c_{\varsigma 4, i}$	LTD4 parameters, $i = \{1, 2, 3, 4\}$

Neural Network Compensation Using Feedback Error Learning

k_0, k_1, k_2	Feedback control gains
$R_{\varsigma 3}$	LTD3 parameter
$c_{\varsigma 3, i}$	LTD3 parameters, $i = \{1, 2, 3\}$
$\mathbf{W}$	Weight matrix of neural network
$\mathbf{w}$	Output layer weight vector of neural network
$\mathbf{V}$	Hidden layer weight matrix of neural network
$\mathbf{x}_N$	Synaptic input vector
P	Number of neurons
η, η_w, η_v	Learning rates

Adaptive Feedforward Compensation Using Neural Networks

L	Number of neurons
v_i	Input weights, $i = \{1, \dots, L\}$
w_i	Output weights, $i = \{1, \dots, L\}$
η_1, η_2	Learning rates
k_P, k_D, k_{2D}	Feedback control gains