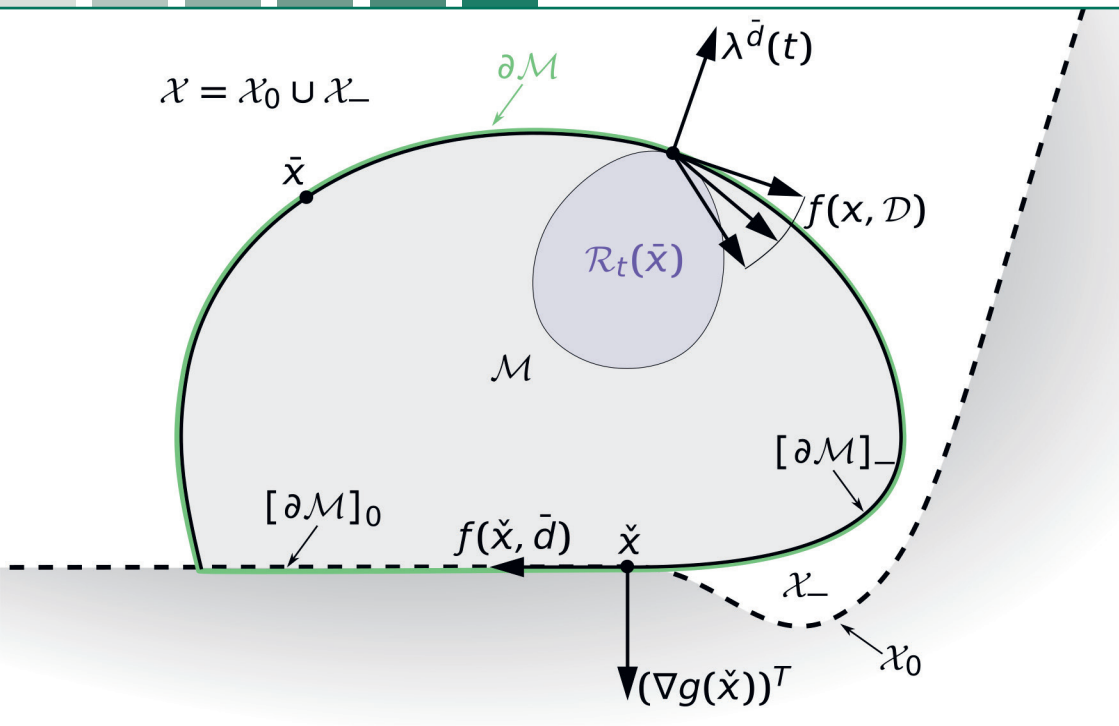
On Invariant Sets and the Theory of Barriers for Constrained Systems and their Applications

Tim Aschenbruck



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To my parents and my sister.

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This thesis is the result of my work at the Automatic Control and System Dynamics Laboratory of the Technische Universität Chemnitz. I would like to express my sincere appreciation to the large number of people who accompanied me during this time.

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Kurzfassung

Invariante Mengen dienen als Grundkonzept zur Formulierung, Analyse und Lösung von Problemstellungen dynamischer Systeme mit stetig steigenden Sicherheitsanforderungen. Diese werden als Systembeschränkungen formuliert, um einen zulässigen Arbeitsbereich des Systemmodells zu definieren, deren Einhaltung durch eine Problemformulierung mittels invarianter Mengen ermöglicht wird.

Diese Arbeit gliedert sich in drei Teile und beschäftigt sich in ihrer Gesamtheit mit invarianten Mengen. Im Besonderen mit robuster und gesteuerter Invarianz und dessen Anwendung zur Lösung verschiedener regelungs- und anwendungsorientierter Problemstellungen mit dem Fokus auf dynamischen Systemen mit Beschränkungen. Hierbei wird ein neuer Ansatz zur sicheren Steuerung mehrerer linearer Systeme ohne individuelle Regler präsentiert sowie ein Beitrag zur Methodenentwicklung für nichtlineare Systeme zur Bestimmung der größten robust positiv invarianten Menge vorgestellt und zur Analyse anwendungsorientierter Problemstellungen eingesetzt.

Der erste Teil der Arbeit betrachtet mehrere lineare Systeme und ein aus der hierarchischen Regelung motiviertes Problem, bei dem die Einhaltung von Beschränkungen zum sicheren Betrieb mittels individueller Regler für jedes System erfolgt. Es wird ein mengenbasierter Ansatz präsentiert, der den sicheren Betrieb mittels einer identischen Stellgröße garantiert, basierend auf einem Regler, der eine Systemaggregation betrachtet. Das aggregierte System wird aus den Einzelsystemen hergeleitet und dient als Repräsentant des aggregierten Systemverhaltens. Abgeleitete mengenbasierte Bedingungen sorgen für die Einhaltung der einzelnen Systembeschränkungen, auf Basis robuster und gesteuerter Invarianzeigenschaften, zur Abschätzung von Trajektorienabweichungen und Sicherstellung der Existenz einer geeigneten Stellgröße.

Der zweite Teil der Arbeit beschäftigt sich mit der Methodenentwicklung zur maximalen robust positiv invarianten Menge für nichtlineare Systeme. Die Methode konzentriert sich auf die Grenze dieser Menge, bestehend aus speziellen Systemtrajektorien, die das Maximumprinzip erfüllen und die Beschränkungen an speziellen Punkten tangential schneiden. Aus der Methode folgt eine exakte Beschreibung der Grenztrajektorien, die mittels Rückwärtsintegration, ausgehend vom Tangentialpunkt auf der Beschränkung, bestimmt werden. Die Methode basiert auf der Barrierentheorie, welche ursprünglich für gesteuert invariante Mengen entwickelt wurde, und erweitert diese um die maximale robust positiv invariante Menge.

Der dritte Teil der Arbeit nutzt die entwickelte Methode und appliziert die Barrierentheorie auf praktische Problemstellungen wie die Analyse der transienten Stabilität von Energieversorgungsnetzen, die Analyse von Räuber-Beute Systemen unter der Prämisse des Erhalts beider Organismen sowie die Analyse von epidemischen Krankheitsmodellen unter beschränkten Infektionszahlen. Die Methode ermöglicht hierbei die Betrachtung der nichtlinearen Systemmodelle sowie die Bestimmung der exakten Grenzen der invarianten Mengen.

Abstract

Set invariance provides a framework to formulate, analyze, and solve control problems related to dynamical systems and their continually increasing requirements on safe operation. A common way to formulate safety conditions for mathematical system models are constraints, which define the allowed space of operation. Constraint satisfaction is, in general, a challenging task in control and can be achieved using invariant sets.

This thesis is concerned with invariant sets, with the focus on robust invariance and control invariance, and presents their beneficial utilization to solve problems in control as well as application areas such as engineering, ecology and epidemiology. In particular, the problem of constrained dynamical systems is considered and a novel set-based approach to safely operate a collection of linear systems without individual control signals is presented. Furthermore, a set-based method is developed to determine the largest robust positively invariant set for nonlinear systems and applied to analyze problems in various application areas. Hence, the thesis can be divided into three main parts.

The first part of this thesis considers a collection of linear systems and a problem mainly motivated from hierarchical control, where constraint satisfaction is usually obtained via individual controllers for each system. A novel approach is presented to guarantee safe control of all systems via aggregating these systems and compute an identical control signal based on the aggregated system that guarantees constraint satisfaction. Conditions for constraint satisfaction are derived using robust invariance to estimate deviations between the dynamical behavior of the considered systems and control invariance to guarantee the existence of an control input to satisfy the conditions. The conditions are formulated as constraints for the aggregated system.

The second part of this thesis is concerned with the development of a method to determine the maximal robust positively invariant set for constrained nonlinear systems. This method focuses on the set's boundary which is made up of special system trajectories that satisfy the maximum principle and intersect the constraint boundary in a tangential manner. The method provides an exact description of these boundary trajectories, which are determined by backwards integration from the identified tangential intersection points on the constraint. This development is based on the theory of barriers, an existing approach in the context of control invariance, and extends this theory to determine the maximal robust positively invariant set.

The third part of this thesis uses the developed method and applies the theory of barriers to practical problems in different application areas. Using this method, enables the analysis of their nonlinear system models and straightforwardly computing the exact set boundaries by integrating the system dynamics. The practical problems are analyzed utilizing robust and control invariant sets and are concerned with the transient stability problem in power grids, the avoidance of species extinction in predator prey systems, and maintaining infection caps for epidemic diseases.

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