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Sakthi Thangavel

Efficient Robust Multi-stage Nonlinear Model Predictive Control Strategies to Handle Plant-model Mismatch

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Chemieingenieurwesen

Efficient Robust Multi-stage Nonlinear Model Predictive Control Strategies to Handle Plant-model Mismatch

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Declaration of authorship

I, Sakthi THANGAVEL, declare that this thesis titled "Efficient Robust Multi-stage Nonlinear Model Predictive Control Strategies to Handle Plant-model Mismatch" and the work presented in it are my own. I confirm that:

- This work was done wholly or mainly while candidature for a research degree at this University.
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- I have acknowledged all primary sources of help.
- Where the thesis is based on work done by myself or jointly with others, I have clarified what was done by others and what I have contributed myself.

Dortmund, December 17, 2023

Previously published material

Parts of the results presented in this work and used in the context of this dissertation were published beforehand. These publications are indicated by references throughout the thesis, and they are listed below with references to the respective chapters. For each publication, Sakthi Thangavel, in the following called *the author*, provides a statement about the extent of the contribution.

The classification of the contribution of the author is defined as

largely – a contribution is classified as *largely*, if most of the work (scientific, technical, and writing) was done (solely) by the author.

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However, notice that scientific work is usually done in a joint effort of a research group, where ideas evolve in discussions and debates about scientific results and methodologies. Thus, a clear distinction between individual contributions is often impossible. The following comments were made to the best knowledge of the author.

Chapter	Statement	Reference(s)
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5	Published in	Thangavel et al. (2018a)
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7	Published in	Thangavel and Engell (2019)
8	Published in	Thangavel et al. (2019b)

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(Thangavel et al., 2020d) Thangavel, S.; Paulen, R.; Engell, S. Robust multi-stage nonlinear model predictive control using sigma points. *Processes* **2020,** *8* (7).

Research, method development, theory, simulation study, and writing were done by the author largely. R. Paulen was involved in the method development partly. All the authors contributed to the writing, revision, and proof reading.

(Thangavel et al., 2018b) Thangavel, S.; Lucia, S.; Paulen, R.; Engell, S. Dual robust nonlinear model predictive control: A multi-stage approach. *Journal of Process Control* **2018,** *72,* 39–51.

Research, method development, theory, simulation study, and writing were done by the author largely. Research, method development, and theory were done by the author during his master thesis partly. R. Paulen was involved in the method development and writing of the method partly. All the authors contributed to the writing, revision, and proof reading.

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(Thangavel et al., 2015) Thangavel, S.; Lucia, S.; Paulen, R.; Engell, S. Towards dual robust nonlinear model predictive control: A multi-stage approach. *American Control Conference (ACC)* **2015,** 428–433.

Research, method development, theory and simulation study were done by the author during his master thesis largely. S. Lucia and R. Paulen were involved in the method development and writing largely. All the authors contributed to the writing, revision, and proof reading.

(Thangavel et al., 2017) Thangavel, S.; Lucia, S.; Paulen, R.; Engell, S. Robust nonlinear model predictive control with reduction of uncertainty via dual control. *21st International Conference on Process Control (PC)* **2017,** 48–53.

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(Thangavel et al., 2020a) Thangavel, S.; Paulen, R.; Engell, S. Adaptive multi-stage NMPC using sigma point principles. *European Control Conference (ECC)* **2020,** 196–201.

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(Thangavel et al., 2020b) Thangavel, S.; Paulen, R.; Engell, S. Dual multi-stage NMPC using sigma point principles. *IFAC-PapersOnLine* **2020,** *53* (2), 21st IFAC World Congress, 11243–11250.

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(Thangavel et al., 2018a) Thangavel, S.; Aboelnour, M.; Lucia, S.; Paulen, R.; Engell, S. Robust dual multi-stage NMPC using guaranteed parameter estimation. *IFAC-PapersOnLine* **2018,** *51* (20), 6th IFAC Conference on Nonlinear Model Predictive Control, 72–77.

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(Thangavel et al., 2018c) Thangavel, S.; Subramanian, S.; Lucia, S.; Engell, S. Handling structural plant-model mismatch using a model-error model in the multi-stage NMPC framework. *IFAC-PapersOnLine* **2018,** *51* (15), 18th IFAC Symposium on System Identification (SYSID), 1074–1079.

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(Thangavel et al., 2019a) Thangavel, S.; Subramanian, S.; Engell, S. Robust NMPC using a model-error model with additive bounds to handle structural plant-model mismatch. *IFAC-PapersOnLine* **2019,** *52* (1), 12th IFAC Symposium on Dynamics and Control of Process Systems, including Biosystems (DYCOPS), 592–597.

Research, method development, theory, simulation study, and writing were done by the author largely. All the authors contributed to the writing, revision, and proof reading.

(Thangavel and Engell, 2019) Thangavel, S.; Engell, S. Handling plant-model mismatch using multi-stage NMPC with model-error model. *22nd International Conference on Process Control* **2019,** 1–6.

Research, method development, theory, simulation study, and writing were done by the author largely. S. Engell contributed to the writing, revision, and proof reading.

(Thangavel and Engell, 2020) Thangavel, S.; Engell, S. An efficient model-error model update strategy for multi-stage NMPC with model-error model. *IFAC-PapersOnLine* **2020,** *53* (2), 21st IFAC World Congress, 7217–7222.

Research, method development, theory, simulation study, and writing were done by the author largely. All the authors contributed to the writing, revision, and proof reading.

(Thangavel et al., 2019b) Thangavel, S.; Subramanian, S.; Paulen, R.; Engell, S. Robust multi-stage NMPC under structural plant-model mismatch without full-state measurements. *18th European Control Conference (ECC)* **2019,** 781–786.

Research, method development, theory, simulation study, and writing were done by the author largely. S. Subramanian involved in the method development partly. All the authors contributed to the writing, revision, and proof reading.

"Change is the only constant in life." Heraclitus

Abstract

The focus of this thesis is to reduce the conservatism introduced by robust nonlinear model predictive controller (NMPC) approaches in the presence of plant model mismatch. Among the robust NMPC schemes, the multi-stage NMPC achieves a low degree of conservatism by incorporating the existence of recourse in its predictions. Therefore, the thesis employs the multi-stage formulation based on scenario trees.

First multi-stage NMPC is improved for parametric uncertainties. The standard approach over-approximates the uncertainty set by a box and generates the scenario tree. If the uncertainty set is not a box, this enlarges the uncertainty set and results in a performance loss. This is mitigated by using sigma points to generate the scenario tree and by computing the future plant evolutions using the unscented transformation. The sigma points capture the true mean and covariance of the uncertainty set and results in a better performance.

Second, adaptive and dual approaches are introduced to improve the performance of multi-stage NMPC in the presence of unknown but time-invariant parameters. The adaptive approach uses plant measurements to estimate the unknown parameters. The dual approach addresses the trade-off between utilizing control inputs for system excitation to ensure accurate parameter estimation and optimizing control actions, resulting in an improved performance.

Third, the existing robust techniques do not address structural plant-model mismatch. The concept of model-error models (MEM), as used in linear control theory to achieve robustness against structural plant-model mismatch, is extended to the robust NMPC framework. The MEM dynamically bounds the uncertainty region around the nominal

model of the plant and a scenario tree is constructed using the nominal and the MEM to address structural plant-model mismatch.

The proposed extensions are evaluated using examples from the chemical and biochemical engineering field, showing a significant improvement to the existing schemes.

Kurzfassung

Das Hauptziel dieser Arbeit ist es, die Konservativität zu reduzieren, die durch robuste NMPC-Ansätze entsteht. Unter den verschiedenen robusten NMPC-Ansätzen erreicht Multi-Stage NMPC einen geringeren Grad an Konservativität, weil die Existenz von Feedback in der Zukunft explizit einbezogen wird.

Zum einen werden parametrische Unsicherheiten betrachtet. Der übliche Multi-Stage-NMPC-Ansatz mit diskreten Szenarien überschätzt die Modellunsicherheit durch eine Box und verwendet einen Szenariobaum aus den Ecken der Box. Wenn die Unbestimmtheitsmenge keine Box ist, vergrößert dies die Konservativität und führt zu einem signifikanten Performanceverlust. Dies wird durch die Verwendung von Sigma-Punkten zur Bildung des Szenariobaums reduziert und die zukünftigen Evolutionen der Anlage werden unter Verwendung der Unscented Transformation berechnet. Die Sigma-Punkte erfassen den wahren Mittelwert und die Kovarianz der Unbestimmtheitsmenge genauer und führen zu einer besseren Reglerperformance.

Zweitens werden adaptive und duale Ansätze eingeführt, um die Performance von Multi-Stage NMPC bei Vorhandensein unbekannter, aber konstanter Parameter zu verbessern. Der adaptive Ansatz nutzt Anlagenmessungen zur Schätzung der unbekannten Parameter. Der duale Ansatz optimiert explizit den Kompromiss zwischen der Anregung der Regelstrecke durch Probing-Aktionen, die zu einer genaueren Schätzung der unbekannten Parameter führen, und optimalen Regeleingaben zur Verbesserung der Regelgüte.

Drittens werden unstrukturierte Modellunsicherheiten betrachtet. Der Ansatz eines Model-error Models (MEM), das in der linearen Regelungstheorie zur Beschreibung von Modellunsicherheiten verwendet wird wird in dieser Arbeit erstmals auf robustes NMPCangewendet. Das MEM grenzt den Unsicherheitsbereich um das nominale Modell der Anlage dynamisch ein. Der Szenariobaum wird unter Verwendung des nominalen und des MEM konstruiert.

Die verschiedenen in dieser Arbeit vorgeschlagenen Erweiterungen werden anhand von Beispielen aus dem Bereich der chemischen und der biochemischen Verfahrenstechnik erprobt und bewertet. Die Simulationsergebnisse zeigen, dass mit den vorgeschlagenen Erweiterungen eine signifikante Leistungsverbesserung gegenüber den bestehenden Verfahren erreicht wird.

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List of abbreviations

A-MS	**A**daptive **M**ulti-**S**tage NMPC
A-MS-BAS	**A**daptive **M**ulti-**S**tage NMPC based on the **B**ox over-**A**pproximation of the reachable set of **S**tates
A-MS-VA-GPE	**A**daptive **M**ulti-**S**tage NMPC based on the **V**ertex over-**A**pproximation of **G**uaranteed **P**arameter **E**stimation solution set
CSTR	**C**ontinuous **S**tirred **T**ank **R**eactor
CV	**C**onstraint **V**iolation
D-MS-ALE	**D**ual **M**ulti-**S**tage NMPC based on **A**pproximate future **L**east-squares **E**stimates
D-MS-BAS	**D**ual **M**ulti-**S**tage NMPC based on the **B**ox over-**A**pproximation of the reachable set of **S**tates
D-MS-OLE	**D**ual **M**ulti-**S**tage NMPC based on **O**ptimistic future **L**east-squares **E**stimates
D-MS-VA-GPE	**D**ual **M**ulti-**S**tage NMPC based on the **V**ertex over-**A**pproximation of **G**uaranteed **P**arameter **E**stimation solution set
D-MS-VLE	**D**ual **M**ulti-**S**tage NMPC based on **V**arying future **L**east-squares **E**stimates
EHE	**E**xternal **H**eat **E**xchanger
EKF	**E**xtended **K**alman **F**ilter
GPE	**G**uaranteed **P**arameter **E**stimation
KKT	**K**arush **K**uhn **T**ucker
LQR	**L**inear **Q**uadratic **R**egulator
MEM	**M**odel-**E**rror **M**odel
MPC	**M**odel **P**redictive **C**ontrol
MPEC	**M**athematical **P**rogram with **E**quilibrium **C**onstraints
MS	standard **M**ulti-**S**tage NMPC
MS-AB	standard **M**ulti-**S**tage NMPC with **A**dapted parameter **B**ounds to handle structural mismatch
MS-BAC	**M**ulti-**S**tage NMPC based on the **B**ox over-**A**pproximation of the reachable set of **C**onstraint function

MS-BAS	**M**ulti-**S**tage NMPC based on the **B**ox over-**A**pproximation of the reachable set of **S**tates
MS-MEM	standard **M**ulti-**S**tage NMPC with **M**odel-**E**rror **M**odel
MS-MEM-CT	**M**ulti-**S**tage NMPC with **M**odel-**E**rror **M**odel based on **C**onstraint **T**ightening
MS-MEM-EMU	**M**ulti-**S**tage NMPC with **M**odel-**E**rror **M**odel using **E**ntire **M**odel-error model **U**pdate
MS-MEM-SE	**M**ulti-**S**tage NMPC with **M**odel-**E**rror **M**odel and **S**tate **E**stimation
MS-MEM-SU	**M**ulti-**S**tage NMPC with **M**odel-**E**rror **M**odel to handle **S**tructured and **U**nstructured uncertainties
MS-SAD	**M**ulti-**S**tage NMPC to handle **S**tructural plant-model mismatch using an **A**dditive **D**isturbance
MS-VA	standard **M**ulti-**S**tage NMPC based on the **V**ertex over-**A**pproximation
NMPC	**N**onlinear **M**odel **P**redictive **C**ontrol
OV	**O**ptimization **V**ariables
T-MS-MEM	**T**ube-enhanced **M**ulti-**S**tage NMPC with **M**odel-**E**rror **M**odel

List of symbols

Mathematical operator

$[a]$	a^{th} element of a vector
$[a, b]$	a^{th} row b^{th} column of a matrix
$\lvert\cdot\rvert$	absolute value of all the elements of a vector
$*$	all elements in a row or column
z	delay block
$\circ$	Hadamard or Schur product
$\boldsymbol{I}^a$	Identity matrix of dimension $a \times a$
∞	infinity
$\in$	is in/ belongs to / is an element of
$(\cdot)^{\frac{1}{2}}$	matrix square root obtained using Cholesky decomposition
max	maximum
min	minimum
$\ominus$	Minkowski/Pontryagin's difference
$\lceil\cdot\rceil$	rounds a number to the next largest integer
$\cup$	set union
$\subset$	strict subset of
$\subseteq$	subset of
^{T}	transpose operation
$\mathbf{1}^a$	vector of ones of length a
$\mathbf{0}^a$	vector of zeros of length a

Special function

$\boldsymbol{h}$	actual model of the plant
$\hat{\boldsymbol{h}}$	actual plant measurement
$\boldsymbol{g}$	constraint function
$\mathscr{D}$	diagonal elements of a matrix
$\mathscr{L}$	eigenvalues of the matrix
$\mathscr{I}$	ellipsoidal over-approximation of intersection between two ellipsoids
$\mathscr{E}$	ellipsoidal parametric or disturbance set definition

$\mathscr{F}$	Fisher distribution
$\mathscr{R}$	half the distance between the two points
$\boldsymbol{f}$	known model of the plant
$\mathscr{M}$	Lagrangian function Jacobian
$\mathscr{P}_{\mathrm{m}}$	measured states in the state vector
L	NMPC stage cost
$\hat{\boldsymbol{f}}$	nominal model measurement
$\mathscr{O}$	predicted future least squares estimates bound
$\mathscr{P}$	projects true plant state to modeled state
$\mathscr{C}_a$	set of all combinations of three vector elements
$\mathscr{C}_{vp}$	set of all combinations of two vector elements
$\mathbb{S}$	sigma-points of the confidence region
$\mathscr{D}^{\frac{1}{2}}$	square roots of the diagonal elements of the matrix.
$\mathscr{P}_{\mathrm{um}}$	unmeasured states in the state vector
$\mathscr{U}$	unscented transformation
$\underline{\mathscr{T}}$	vector elements with maximum values of two vectors
$\overline{\mathscr{T}}$	vector elements with minimum values of two vectors
$\mathscr{V}_{\mathrm{sp}}$	vertices of box-approximation of confidence region partition

Set representation

$\mathbb{I}_g$	constraint indices
$\mathcal{D}$	continuous uncertainty set
$\mathbb{W}$	disturbance continuous set
$\mathbb{U}$	input constraint continuous set
$\mathbb{I}$	integer numbers set
$\mathcal{R}^{a\times b}$	matrix set of real numbers with a rows and b columns
$\mathbb{I}_s$	measurement indices
$\mathbb{I}_b(k)$	nodes at stage k in the scenario tree
$\mathbb{I}_{N_m}$	offline measurement indices
$\mathcal{R}$	real numbers set
$\mathbb{I}_{st}$	scenario tree indices
$\mathbb{I}_{sp}$	sigma point indices
$\mathbb{X}$	state constraint continuous set
$\mathbb{I}_{n_x}$	states indices
$\mathbb{V}$	tightened control set
$\tilde{\mathbb{X}}$	tightened state constraint set
$\mathbb{I}_{ar}$	time indices after the robust horizon
$\mathbb{I}_{br}$	time indices until the robust horizon
$\mathbb{I}_{n_d}$	uncertainty indices
$\mathbb{D}$	uncertainty realizations considered in the scenario tree

$\mathbb{X}$	vertices of the box over-approximation of the reachable state set and state mean
$\mathbb{I}_{2^n d}$	vertices of uncertainty box over-approximation indices

Notation

$\boldsymbol{z}$	actual plant states
$\mathfrak{P}$	a-posteriori estimation error covariance matrix
$\hat{\mathfrak{P}}^{-}$	a-priori estimation error covariance matrix
$\check{\boldsymbol{g}}$	constraint function value
$\boldsymbol{u}$	control input
s	current NMPC iteration
t	current time
k	discrete time step
$\boldsymbol{e}^{\mathrm{m}}$	error between the plant dynamics and the prediction
$\boldsymbol{F}$	Fisher matrix
γ	induced l_∞ gain of the static nonlinear operator
$\mathfrak{K}$	Kalman gain
$\boldsymbol{l}_{=}$	Lagrange equality multipliers
$\boldsymbol{l}_{><}$	Lagrange inequality multipliers
$\boldsymbol{B}$	linear control matrix
$\boldsymbol{B}^e$	linear model-error model control dynamics
$\boldsymbol{e}$	linear model-error model dynamics
$\boldsymbol{A}^e$	linear model-error model system dynamics
$\boldsymbol{A}$	linear system matrix
$\bar{\boldsymbol{l}}$	magnitude of the measurement noise lower and upper bound
$\overline{\boldsymbol{w}}$	magnitude of the model-error model additive mismatch term's lower and upper bound
$\boldsymbol{l}$	measurement noise
$\boldsymbol{L}$	measurement noise covariance matrix
$\underline{\boldsymbol{l}}$	measurement noise lower bound
$\bar{\boldsymbol{l}}$	measurement noise upper bound
$\boldsymbol{x}$	modeled system states
$\boldsymbol{w}$	model-error model additive mismatch term
ω_n	Nyquist frequency
$\star$	optimal value for optimization problem's decision variable
ϕ	parameter for computing ellipsoid over-approximating intersection between two ellipsoid
χ	parameter for computing ellipsoid over-approximating Minkowski sum of ellipsoids
$\boldsymbol{x}^m$	plant full state measurement
$\boldsymbol{y}^m$	plant measurements

$\underline{x}_m$	reachable state set lower bound
$\overline{x}_m$	reachable state set upper bound
t_s	sampling time of the plant
j	scenario index
$\boldsymbol{s}$	sensitivity of parameters
$\boldsymbol{w}$	slack variables
$\mathfrak{Q}$	state noise covariance matrix
$\hat{\boldsymbol{d}}$	subset of uncertainty classified as large uncertainty
$\boldsymbol{H}$	transfer function of linear model obtained using offline data
$\hat{\boldsymbol{H}}$	transfer function of linear model obtained using online data
$\boldsymbol{v}$	tube-enhanced multi-stage/tube-based NMPC primary controller control input
$\tilde{\boldsymbol{x}}$	tube-enhanced multi-stage/tube-based NMPC primary controller nominal model state
$\boldsymbol{P}$	uncertainty covariance matrix
$\boldsymbol{d}$	uncertainty in the system
$\tilde{\boldsymbol{P}}$	uncertainty initial covariance matrix
$\underline{\tilde{\boldsymbol{d}}}$	uncertainty initial lower bound
$\overline{\tilde{\boldsymbol{d}}}$	uncertainty initial upper bound
$\underline{\boldsymbol{d}}$	uncertainty lower bound
$\underline{\mathfrak{d}}$	uncertainty lower bound a-priori estimate
$\tilde{\boldsymbol{d}}$	uncertainty nominal value
$\overline{\boldsymbol{d}}$	uncertainty upper bound
$\overline{\mathfrak{d}}$	uncertainty upper bound a-priori estimate
$\boldsymbol{\Delta}$	unknown static nonlinear mapping vector
δ	unknown static nonlinear mapping
$\boldsymbol{G}_c$	unscented transformation constraint covariance matrix
$\boldsymbol{g}_m$	unscented transformation constraint mean
$\boldsymbol{X}_c$	unscented transformation state covariance matrix
$\boldsymbol{x}_m$	unscented transformation state mean
$\boldsymbol{e}^{\mathrm{se}}$	worst-case error between the prediction and state estimate confidence region

Dimension (number of)

n_z	actual plant states
n_g	additional constraints considered in the NMPC
n_u	control inputs
n_y	measurements
n_x	modeled states
n_d	modeled uncertain variables
n_t	total nodes in the scenario tree

n_b	uncertainty realization at each node
$n_{\boldsymbol{A}^e\boldsymbol{B}^e}$	unknown parameters present in $\boldsymbol{A}^e$ and $\boldsymbol{B}^e$

Tuning parameters

$\boldsymbol{R}_{ac}$	ancillary controller control weighting matrix
$\boldsymbol{Q}_{ac}$	ancillary controller states weighting matrix
ξ	confidence level of Gaussian distribution
$\boldsymbol{R}$	control inputs weighting matrix
$\xi_{\boldsymbol{y}}$	measurements confidence level of Gaussian distribution
$\boldsymbol{Q}_{\boldsymbol{w}}$	model-error model weighting matrix
N_p	prediction horizon length
N_r	robust horizon length
β	scaling factor increase factor
$\kappa_{\boldsymbol{g}}$	scaling factor of MS-BAC NMPC
$\kappa_{\boldsymbol{x}}$	scaling factor of MS-BAS NMPC
w_{tol}	slack variables tolerance
$\xi_{\boldsymbol{x}}$	state estimates confidence level
$\boldsymbol{Q}$	states weighting matrix
$\boldsymbol{Q}_{N_h}$	terminal state weighting matrix
N_h	time horizon of finite horizon LQR
$\boldsymbol{q}_{\text{EKF}}$	weighting factor for tuning EKF
θ	weights for each scenario
$\boldsymbol{v}$	weights for the sigma points

Notes on the notation:

Throughout this thesis capital bold letters denote matrices, e.g., matrix $\mathbf{A}$, lower case bold letters denote vectors, e.g., vector $\mathbf{b}$, lower case bold letters followed by arguments inside brackets denote vector-valued function, e.g., function $\mathbf{f}(\cdots)$, and function $\mathbf{f}_{[a]}(\cdots)$ represents a^{th} element of the vector-valued function.

For inequalities the symbols $\leq, <, \geq$, and $>$ are used, where in the notation $\mathbf{a} \leq \mathbf{b}$ the operation is defined as an element-wise comparison.